

Example #19 – Intermediate Dynamics: Principle of Virtual Work (Simple Mechanism)

Given:

$$L_{AD} = L_{CE} = 0.2 \text{ (m)}; \quad L_{DB} = L_{EB} = 0.4 \text{ (m)}$$

Active Forces:

$$\text{Weight forces: } m_{AB} = m_{CB} = m = 4 \text{ (kg)}$$

$$\text{Spring: } k = 800 \text{ (N/m)}; \quad \ell_u = 0.15 \text{ (m)}$$

Find:

$$y_{EQ} \text{ the equilibrium position of } C \text{ relative to } A$$

Solution: (using y as the **generalized coordinate**)

For equilibrium, the **principle of virtual work** states

$$(F_y)_{W_1} + (F_y)_{W_2} + (F_y)_{\text{spring}} = 0$$

where

$$(F_y)_{W_1} = -\frac{\partial V_1}{\partial y} = -\frac{\partial}{\partial y}(-mg(\frac{1}{4}y)) = \frac{1}{4}mg$$

$$(F_y)_{W_2} = -\frac{\partial V_2}{\partial y} = -\frac{\partial}{\partial y}(-mg(\frac{3}{4}y)) = \frac{3}{4}mg$$

$$(F_y)_{\text{spring}} = -\frac{\partial V_{\text{spring}}}{\partial y} = -\frac{\partial}{\partial y}\left(\frac{1}{2}k\left(\frac{2}{3}y - 0.15\right)^2\right) = -\frac{1}{2}k(2)\left(\frac{2}{3}y - 0.15\right)\left(\frac{2}{3}\right) = -\frac{2}{3}k\left(\frac{2}{3}y - 0.15\right)$$

Substituting into the principle of virtual work gives

$$F_y = \frac{1}{4}mg + \frac{3}{4}mg - \frac{2}{3}k\left(\frac{2}{3}y_{EQ} - 0.15\right) = mg - \frac{2}{3}k\left(\frac{2}{3}y_{EQ} - 0.15\right) = 0$$

$$\Rightarrow mg = \frac{2}{3}k\left(\frac{2}{3}y_{EQ} - 0.15\right) \Rightarrow y_{EQ} = \frac{3}{2}\left(\left(\frac{3mg}{2k}\right) + 0.15\right) \approx 0.335 \text{ (m)}$$

