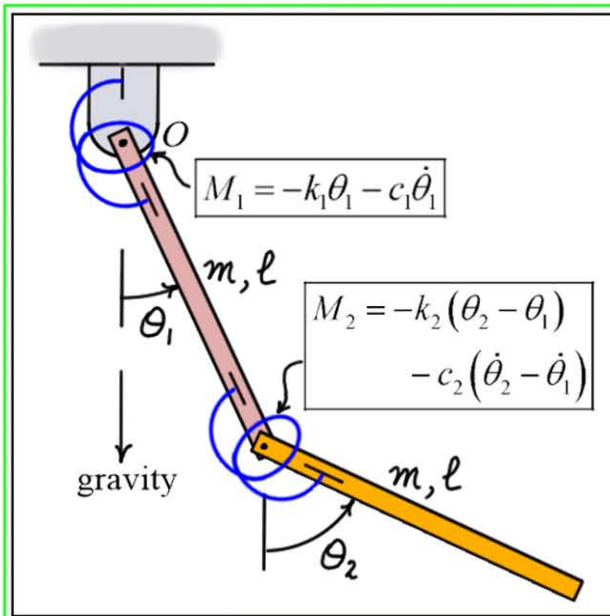
Example #22 – Intermediate Dynamics: Lagrange's Equations (2 DOF system)

Given:

$$M_1 = -k_1\theta_1 - c_1\dot{\theta}_1$$
$$M_2 = -k_2(\theta_2 - \theta_1) - c_2(\dot{\theta}_2 - \dot{\theta}_1)$$



Active Forces/Torques:

$\underset{\sim}{M}_1$, $\underset{\sim}{M}_2$, weight forces

Find:

- ***differential equations of motion*** of the system

Solution: (using θ_1 and θ_2 as generalized coordinates)

Previous Results:

$$\left(\tfrac{4}{3}m\ell^2\right)\ddot{\theta}_1 + \left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_2 - \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_2^2 + \tfrac{3}{2}mg\ell S_1 = M_1(t) - M_2(t)$$

$$\left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_1 + \left(\tfrac{1}{3}m\ell^2\right)\ddot{\theta}_2 + \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_1^2 + \tfrac{1}{2}mg\ell S_2 = M_2(t)$$

Substituting for the torques $M_1(t)$ and $M_2(t)$ gives:

$$\left(\tfrac{4}{3}m\ell^2\right)\ddot{\theta}_1 + \left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_2 - \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_2^2 + \tfrac{3}{2}mg\ell S_1 = -k_1\theta_1 - c_1\dot{\theta}_1 + k_2(\theta_2 - \theta_1) + c_2(\dot{\theta}_2 - \dot{\theta}_1)$$

$$\left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_1 + \left(\tfrac{1}{3}m\ell^2\right)\ddot{\theta}_2 + \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_1^2 + \tfrac{1}{2}mg\ell S_2 = -k_2(\theta_2 - \theta_1) - c_2(\dot{\theta}_2 - \dot{\theta}_1)$$

or

$$\left(\tfrac{4}{3}m\ell^2\right)\ddot{\theta}_1 + \left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_2 - \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_2^2 + \tfrac{3}{2}mg\ell S_1 + (c_1 + c_2)\dot{\theta}_1 - c_2\dot{\theta}_2 + (k_1 + k_2)\theta_1 - k_2\theta_2 = 0$$

$$\left(\tfrac{1}{2}m\ell^2 C_{2-1}\right)\ddot{\theta}_1 + \left(\tfrac{1}{3}m\ell^2\right)\ddot{\theta}_2 + \left(\tfrac{1}{2}m\ell^2 S_{2-1}\right)\dot{\theta}_1^2 + \tfrac{1}{2}mg\ell S_2 + c_2(\dot{\theta}_2 - \dot{\theta}_1) + k_2(\theta_2 - \theta_1) = 0$$

As before, these equations represent a *coupled* set of nonlinear, second-order, ordinary differential equations of motion.