

**ME 2580 Dynamics**  
**Curvilinear Motion – Rectangular Components**

**General Concepts:**

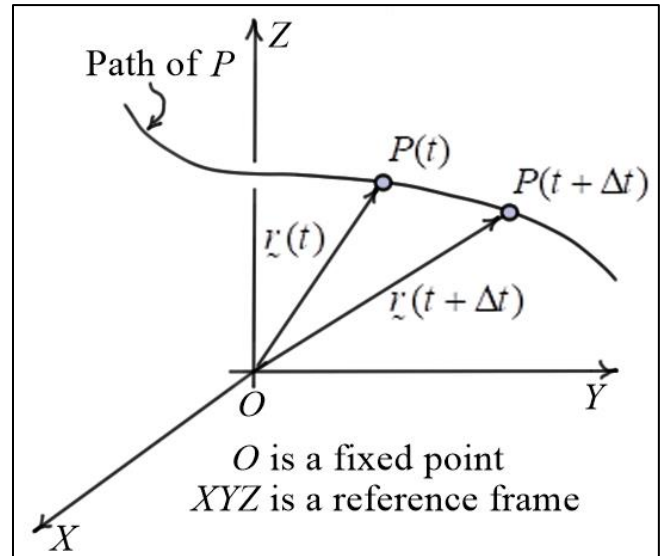
**Position, Velocity, and Acceleration**

If a particle does not move in a straight line, then its motion is said to be *curvilinear*. Given  $\underline{r}(t)$  the position vector of a particle  $P$ , the velocity and acceleration of  $P$  are defined to be

$$\underline{v} = \frac{d\underline{r}}{dt} \quad \text{and} \quad \underline{a} = \frac{d\underline{v}}{dt}.$$

The *velocity*  $\underline{v}$  is *always tangent to the path* of  $P$ .

The *acceleration*  $\underline{a}$  is generally not tangent to the path.



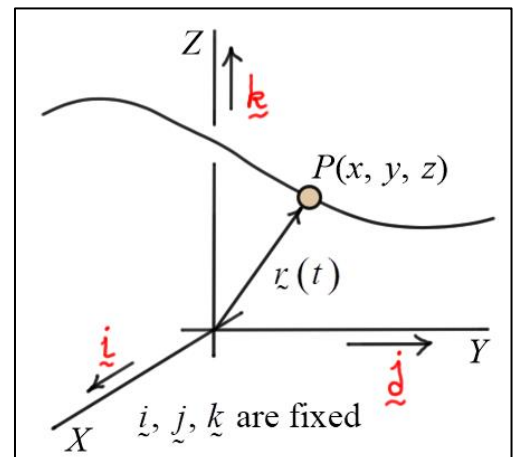
**Rectangular Components**

If we use rectangular components, then the position, velocity, and acceleration vectors may be written as

$$\underline{r}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{k}$$

$$\underline{v}(t) = \dot{x}(t)\underline{i} + \dot{y}(t)\underline{j} + \dot{z}(t)\underline{k}$$

$$\underline{a}(t) = \ddot{x}(t)\underline{i} + \ddot{y}(t)\underline{j} + \ddot{z}(t)\underline{k}$$



**Note** that the methods for straight-line (rectilinear) motion can be applied in each direction.

## Example: The Projectile Problem

If we *neglect air resistance*, the motion of a projectile can be analyzed using the equations for constant acceleration. The *horizontal motion* (X-direction) occurs at a *constant velocity*, and the *vertical motion* (Y-direction) occurs at a *constant acceleration*. The equations that apply in the X and Y directions are

X-direction: (constant velocity,  $v_{x_0}$ )

$$x(t) = x_0 + v_{x_0} t$$

Y-direction: (constant acceleration,  $-g$ )

$$v_y(t) = v_{y_0} - gt \quad y(t) = y_0 + v_{y_0} t - \frac{1}{2} gt^2 \quad v_y^2 = v_{y_0}^2 - 2g(y - y_0)$$

## Derivative of a Rotating Unit Vector in Two Dimensions

In later notes on curvilinear motion, *rotating unit vectors* will be used instead of *fixed unit vectors*. To this end, consider the rotating unit vector  $\underline{e}$  shown in the diagram. It can be written in terms of the fixed unit vectors  $\underline{i}$  and  $\underline{j}$  as

$$\underline{e} = \cos(\theta)\underline{i} + \sin(\theta)\underline{j}$$

Here, angle  $\theta$  varies with time as  $\underline{e}$  rotates. *Differentiating* with respect to time using the chain rule gives

$$\dot{\underline{e}} \triangleq \frac{d\underline{e}}{dt} = -\dot{\theta}\sin(\theta)\underline{i} + \dot{\theta}\cos(\theta)\underline{j} = \dot{\theta}(-\sin(\theta)\underline{i} + \cos(\theta)\underline{j}) \quad \Rightarrow \quad \boxed{\dot{\underline{e}} = \dot{\theta}\underline{e}_\perp}$$

The unit vector  $\underline{e}_\perp$  is rotated 90 degrees counterclockwise from  $\underline{e}$ . Using the vector cross product, we note that  $\underline{e}_\perp$  can be written as  $\underline{e}_\perp = \underline{k} \times \underline{e}$ . Using this observation, write

$$\dot{\underline{e}} = \dot{\theta}(\underline{k} \times \underline{e}) = (\dot{\theta}\underline{k}) \times \underline{e} \quad \Rightarrow \quad \boxed{\dot{\underline{e}} = \underline{\omega} \times \underline{e}}$$

Here,  $\underline{\omega} \triangleq \dot{\theta}\underline{k}$  is the *angular velocity* of the rotating unit vector set  $(\underline{e}, \underline{e}_\perp)$ .

