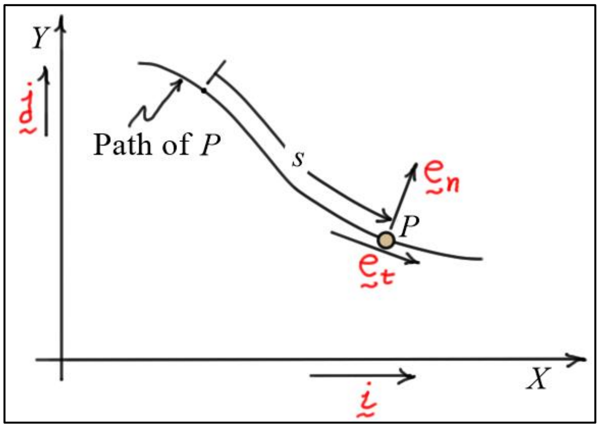
Elementary Dynamics
Curvilinear Motion – Normal and Tangential Components

Normal and Tangential Components

Normal and tangential components refer to components that are ***normal*** and ***tangential*** to the path of *P*. These directions are defined by the unit vectors $\underset{\sim}{e}_n$ and $\underset{\sim}{e}_t$, respectively. Note that they are different from the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ in that their ***orientation changes with time***.



Using normal and tangential components, the ***velocity*** of *P* can be written as

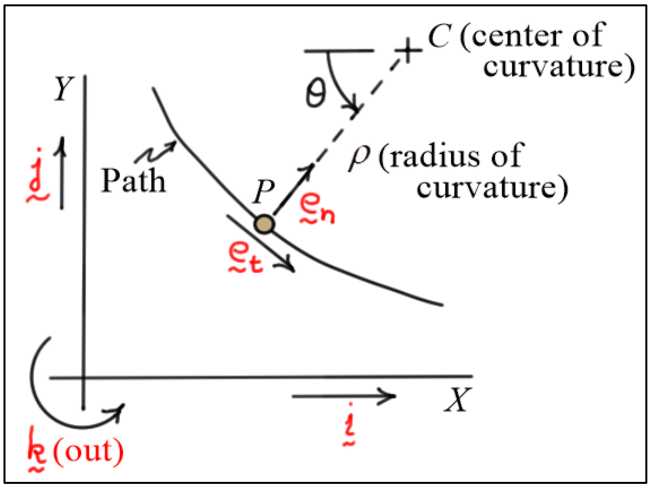
$$\boxed{\underset{\sim}{v} = v\underset{\sim}{e}_t = \frac{ds}{dt}\underset{\sim}{e}_t = \dot{s}\,\underset{\sim}{e}_t}\,.$$

The ***acceleration*** of *P* is found by differentiating the velocity with respect to time (using the product rule)

$$\underset{\sim}{a} = \dot{v}\underset{\sim}{e}_t + v\dot{\underset{\sim}{e}}_t = \dot{v}\underset{\sim}{e}_t + v\left(\dot{\theta}\underset{\sim}{k} \times \underset{\sim}{e}_t\right) = \dot{v}\underset{\sim}{e}_t + v\dot{\theta}\underset{\sim}{e}_n$$

From calculus we know that $\dot{s}$ can be related to $\dot{\theta}$ using ρ the radius of curvature. Specifically,

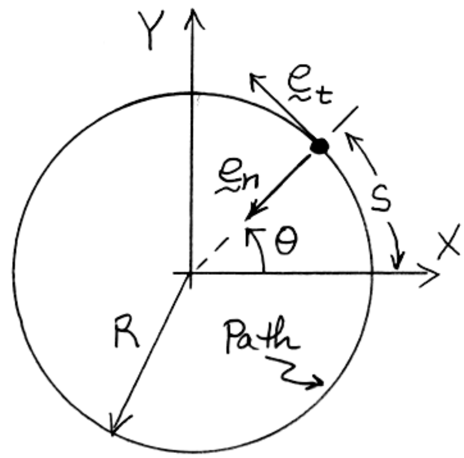
$$v = \rho\dot{\theta} \quad \text{or} \quad \dot{\theta} = v/\rho$$



Substituting this result into the acceleration gives the final result

$$\boxed{\underset{\sim}{a} = \dot{v}\underset{\sim}{e}_t + \left(\frac{v^2}{\rho}\right)\underset{\sim}{e}_n} \qquad \begin{cases} a_t = \dot{v} = \dfrac{dv}{dt} & \text{(tangential acceleration)} \\ a_n = \dfrac{v^2}{\rho} & \text{(normal acceleration)} \end{cases}$$

Special Case: Circular Motion



In the special case of ***circular motion***, we have

$$s = R\theta$$

where θ is measured in radians. ***Differentiating*** with respect to time gives

$$\dot{s} = R\dot{\theta} = R\omega \quad \text{and} \quad \ddot{s} = R\ddot{\theta} = R\alpha \,.$$

Substituting these results into the ***velocity*** and ***acceleration*** formulas from above gives

$$\boxed{\underset{\sim}{v} = \dot{s}\,\underset{\sim}{e}_t = R\dot{\theta}\,\underset{\sim}{e}_t} \quad \text{and} \quad \boxed{\underset{\sim}{a} = \ddot{s}\,\underset{\sim}{e}_t + \left(\frac{\dot{s}^2}{R}\right)\underset{\sim}{e}_n = R\ddot{\theta}\,\underset{\sim}{e}_t + R\dot{\theta}^2\,\underset{\sim}{e}_n = R\alpha\underset{\sim}{e}_t + R\omega^2\underset{\sim}{e}_n}$$