

Elementary Dynamics

Curvilinear Motion – Radial and Transverse Components

Radial and Transverse Components

Another way to describe the motion of P as it moves along a curved path is to use *radial* and *transverse* components. Here, we define the unit vector \underline{e}_r to point radially outward from O to P . The *position vector* of P can then be written as

$$\underline{r} = r \underline{e}_r$$

To find an expression for the *velocity* of P , we differentiate (using the product rule)

$$\underline{v} = \dot{r} \underline{e}_r + r \dot{\underline{e}}_r = \dot{r} \underline{e}_r + r (\dot{\theta} \underline{k} \times \underline{e}_r) = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta \Rightarrow \underline{v} = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta$$

To find an expression for the *acceleration* of P , differentiate again using the product rule.

$$\underline{a} = \ddot{r} \underline{e}_r + \dot{r} \dot{\underline{e}}_r + \dot{r} \dot{\theta} \underline{e}_\theta + r \ddot{\theta} \underline{e}_\theta + r \dot{\theta} \dot{\underline{e}}_\theta$$

Here,

$$\dot{\underline{e}}_r = \dot{\theta} \underline{k} \times \underline{e}_r = \dot{\theta} \underline{e}_\theta \quad \text{and} \quad \dot{\underline{e}}_\theta = \dot{\theta} \underline{k} \times \underline{e}_\theta = -\dot{\theta} \underline{e}_r$$

Using these two results in the expression for the acceleration and collecting terms gives

$$\underline{a} = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta$$

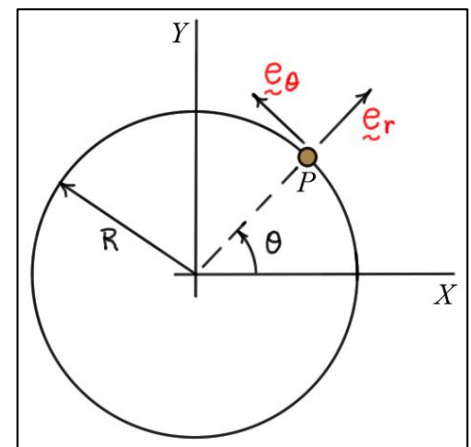
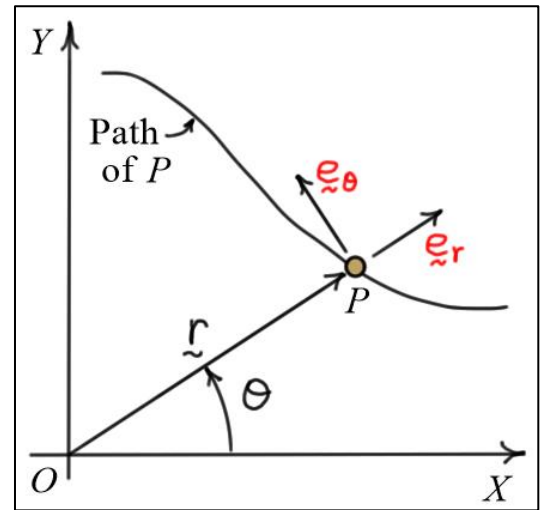
Special Case: Circular Motion

During circular motion the distance between O and P remains *constant* and the transverse unit vector \underline{e}_θ is *tangent* to the path.

$$r = R = \text{constant} \Rightarrow \dot{r} = \ddot{r} = 0$$

$$\Rightarrow \underline{v} = R \dot{\theta} \underline{e}_\theta$$

$$\underline{a} = (-R \dot{\theta}^2) \underline{e}_r + (R \ddot{\theta}) \underline{e}_\theta$$



Cylindrical Components

Cylindrical components represent the extension of the concept of *radial* and *transverse* components to three dimensions. The projection of P onto the XY plane is tracked with radial and transverse directions in that plane. Motion of P perpendicular to the XY plane is tracked by the Cartesian (rectangular) coordinate z . In this case, write

$$\underline{r} = r \underline{e}_r + z \underline{k}$$

$$\underline{v} = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta + \dot{z} \underline{k}$$

$$\underline{a} = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \underline{e}_\theta + \ddot{z} \underline{k}$$

