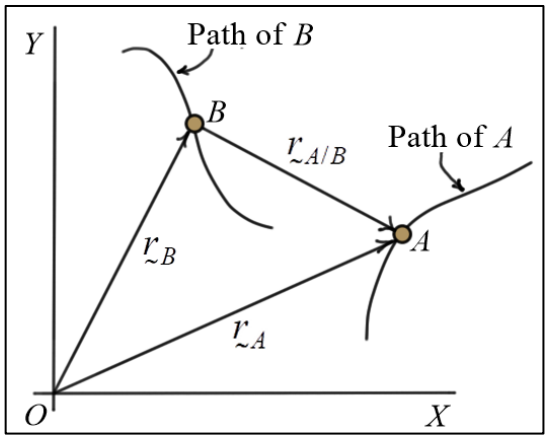
Elementary Dynamics
Relative Motion of Two Particles

The figure shows the paths of motion of two particles *A* and *B*. The vectors $\underset{\sim}{r}_A$ and $\underset{\sim}{r}_B$ represent the ***position vectors*** of *A* and *B* relative to a fixed point *O*, and the vector $\underset{\sim}{r}_{A/B}$ represents the position vector of *A* ***relative*** to *B* (or *A* with respect to *B*). Note that $\underset{\sim}{r}_{A/B}$ starts at *B* and ends at *A*.



Sometimes it is convenient to express the motion of a point relative to another moving point. For example, to describe the motion of particle *A* relative to particle *B*, first note that

$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}$ or $\boxed{\underset{\sim}{r}_{A/B} = \underset{\sim}{r}_A - \underset{\sim}{r}_B}$.

Differentiating this expression gives rise to the definitions of the terms "***relative velocity***" and "***relative acceleration***."

$\boxed{\underset{\sim}{v}_{A/B} = \dot{\underset{\sim}{r}}_{A/B} = \underset{\sim}{v}_A - \underset{\sim}{v}_B}$ "relative velocity – velocity of *A* relative to *B*"

$\boxed{\underset{\sim}{a}_{A/B} = \ddot{\underset{\sim}{r}}_{A/B} = \underset{\sim}{a}_A - \underset{\sim}{a}_B}$ "relative acceleration – acceleration of *A* relative to *B*"

In words, the motion (velocity or acceleration) of *A* relative to *B* represents the ***motion*** of *A* as seen by an ***observer translating*** with *B*. This concept is used extensively in the analysis of rigid body kinematics.