

## Elementary Dynamics

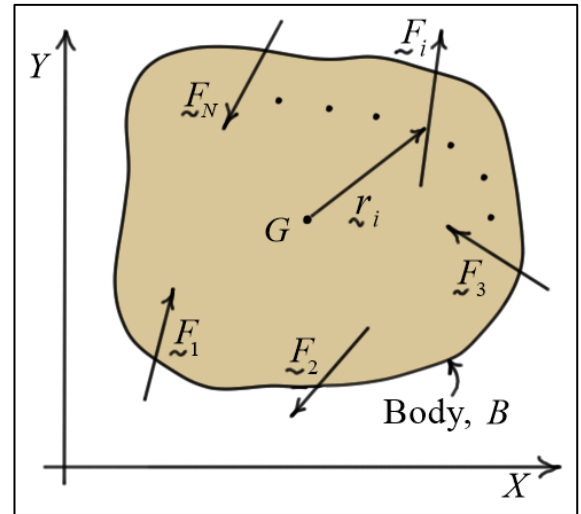
### Principle of Impulse and Momentum for Rigid Body Motion in Two Dimensions

#### General Plane Motion

The figure shows a rigid body moving in two dimensions. The motion is caused by a series of  $N$  forces  $F_i$  ( $i=1, \dots, N$ ). Generally, each force has the effect of **translating** and **rotating** the body. Newton's laws of translational and rotational motion are

$$\sum_{i=1}^N F_i = m a_G = m \left( \frac{dv_G}{dt} \right) = \frac{d}{dt} (m v_G)$$

$$\sum_{i=1}^N (M_G)_i = \sum_{i=1}^N (r_i \times F_i) = I_G \alpha = I_G \left( \frac{d\omega}{dt} \right) = \frac{d}{dt} (I_G \omega)$$



Note here that  $(M_G)_i$  represents the **moment** of force  $F_i$  about the mass center  $G$ . Also, recall that  $I_G$  represents the **mass moment of inertia** of the body about a  $Z$  axis passing through the mass center  $G$ .

The above equations can be **integrated with respect to time** to give

$$\boxed{(m v_G)_1 + \int_{t_1}^{t_2} \left( \sum_{i=1}^N F_i \right) dt = (m v_G)_2} \quad (\text{Principle of Linear Impulse \& Momentum})$$

$$\boxed{(I_G \omega)_1 + \int_{t_1}^{t_2} \left( \sum_{i=1}^N (r_i \times F_i) \right) dt = (I_G \omega)_2} \quad (\text{Principle of Angular Impulse \& Momentum})$$

The **principle of linear impulse and momentum** states that the linear impulses applied to the body over the **time interval**  $t_1 \rightarrow t_2$  give rise to a **change** in the linear momentum of the body.

The **principle of angular impulse and momentum** states that the angular impulses applied to the body over the **time interval**  $t_1 \rightarrow t_2$  give rise to a **change** in the angular momentum of the body.

Note the linear momentum is often written as  $L = m v_G$ , and the angular momentum about  $G$  the mass center as  $H_G = I_G \omega$ .

Note that, like Newton's laws of translational and rotational motion, the above equations are **vector equations**. For planar motion, there are **two scalar linear momentum equations** ( $X$  and  $Y$  directions) and **one scalar angular momentum equation** ( $Z$  direction).

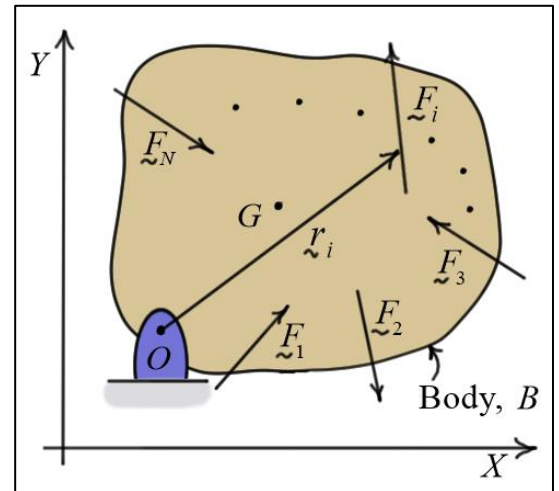
## Special Case

### Fixed Axis Rotation

When a body is undergoing *fixed axis rotation* as shown, the principle of angular momentum can be written about the fixed-point  $O$  as

$$\boxed{(I_O \omega)_1 + \int_{t_1}^{t_2} \left( \sum_{i=1}^N (\mathbf{r}_i \times \mathbf{F}_i) \right) dt = (I_O \omega)_2}$$

In this case,  $\sum_{i=1}^N (\mathbf{r}_i \times \mathbf{F}_i)$  represents the moment of all the forces about the fixed point  $O$ , and  $I_O$  represents the mass moment of inertia of the body about a  $Z$  axis passing through  $O$ .



### Principle of Angular Momentum about an Arbitrary Fixed Point

Consider a rigid body undergoing planar motion. The principle of angular impulse and momentum can also be written relative to an arbitrary fixed-point  $O$  as follows

$$\boxed{(\mathbf{H}_O)_1 + \int_{t_1}^{t_2} \left( \sum_{i=1}^N (\mathbf{r}_i \times \mathbf{F}_i) \right) dt = (\mathbf{H}_O)_2}$$

Here,

$$\boxed{\mathbf{H}_O = I_G \omega + (\mathbf{r}_G \times m \mathbf{v}_G)}$$

Note that  $\mathbf{H}_O$  is the sum of the angular momentum of the body about  $G$  and the moment of the linear momentum about  $O$ . The linear momentum  $\mathbf{L} = m \mathbf{v}_G$  is assumed to have a line of action through  $G$ .

