

Elementary Dynamics

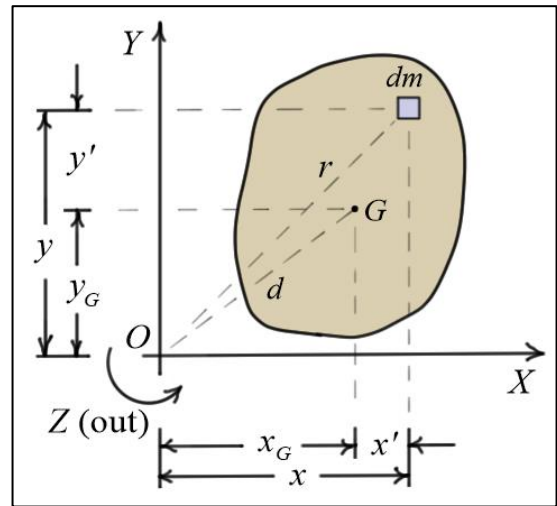
Mass Moments of Inertia in Two Dimensions

Definition of Mass Moment of Inertia

The figure depicts a rigid body in two dimensions. The moment of inertia of the body about the Z -axis passing through the point O is defined as

$$(I_Z)_O = \int_B r^2 dm = \int_B (x^2 + y^2) dm$$

The moment of inertia $(I_Z)_O$ measures the distribution of mass about the Z -axis.



The **larger** the inertia, the **further** the mass is spread away from the axis. The **smaller** the inertia, the **closer** the mass is located to the axis. The units of I_Z are **slug-ft²** or **kg-m²**.

Parallel Axes Theorem

The moment of inertia of the body about the Z -axis can be related to the moment of inertia about an axis parallel to Z and passing through the mass center G as follows:

$$\begin{aligned} (I_Z)_O &= \int_B (x^2 + y^2) dm = \int_B ((x_G + x')^2 + (y_G + y')^2) dm \\ &= \int_B (x_G^2 + y_G^2) dm + \int_B 2x_G x' dm + \int_B 2y_G y' dm + \int_B (x'^2 + y'^2) dm \\ &= (x_G^2 + y_G^2) \int_B dm + 2x_G \underbrace{\left(\int_B x' dm \right)}_{\substack{\text{zero by} \\ \text{definition} \\ \text{of mass center}}} + 2y_G \underbrace{\left(\int_B y' dm \right)}_{\substack{\text{zero by} \\ \text{definition} \\ \text{of mass center}}} + (I_Z)_G \\ &= M d^2 + (I_Z)_G \end{aligned}$$

or

$$(I_Z)_O = (I_Z)_G + M d^2$$

or

$$I_O = I_G + M d^2$$

(Parallel Axes Theorem)

Mass Moments of Inertia for Composite Shapes

The mass moment of inertia of a composite shape is the sum of the inertias of the individual component shapes. If G is the mass center of the composite shape, then

$$I_G = \sum_i (I_G)_i$$

where $(I_G)_i$ represents the moment of inertia of the i^{th} component shape about the composite mass center G and can be calculated as follows

$$(I_G)_i = (I_{G_i}) + m_i d_i^2$$

Here, I_{G_i} represents the moment of inertia of the i^{th} component shape about its own mass center G_i and can be found in the tables for common geometric shapes.

