

Elementary Dynamics

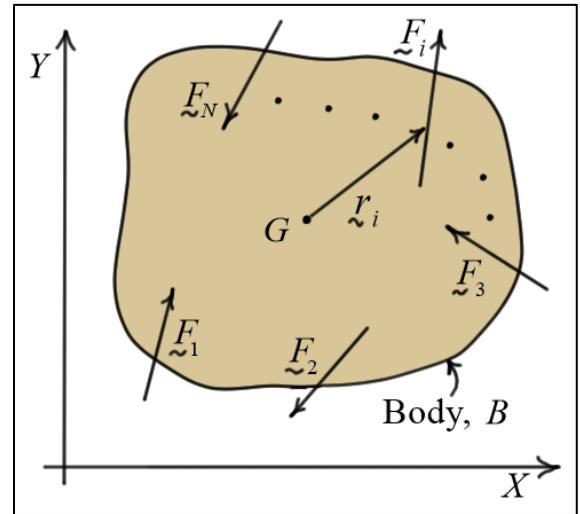
Newton's Law for Rigid Body Motion in Two Dimensions

General Plane Motion

The figure depicts a rigid body moving in two dimensions. The motion is caused by a series of N forces \underline{F}_i ($i=1, \dots, N$). Generally, each force has the effect of both translating and rotating the body. Newton's laws of translational and rotational motion are

$$\sum_i \underline{F}_i = m \underline{a}_G$$

$$\sum_i (\underline{M}_G)_i = \sum_i (\underline{r}_i \times \underline{F}_i) = I_G \alpha$$

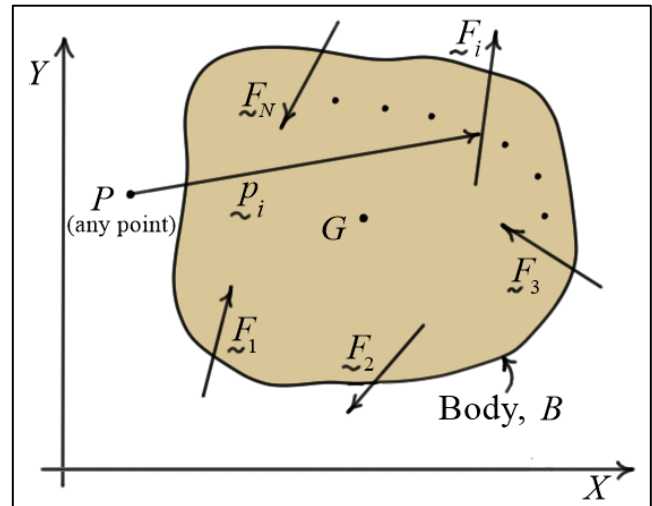


Here, $(\underline{M}_G)_i$ represents the **moment** of force \underline{F}_i about the **mass center** G , and I_G represents the **moment of inertia** of the body about a Z axis passing through G .

Note the equation for rotational motion as written in the boxed equation requires that moments be taken about the mass center G . If moments are to be taken about any point P other than G , Newton's laws of translational and rotational motion are written as

$$\sum_i \underline{F}_i = m \underline{a}_G$$

$$\sum_i (\underline{M}_P)_i = \sum_i (\underline{p}_i \times \underline{F}_i) = I_G \alpha + (\underline{r}_{G/P} \times m \underline{a}_G)$$



Note the term $\underline{r}_{G/P} \times m \underline{a}_G$ is added to the right side of the moment equation and represents the moment of vector $m \underline{a}_G$ about point P . The line of action of $m \underline{a}_G$ is assumed to pass through G .

Special Cases

Pure Translational Motion

When a body is undergoing pure translational motion, the equations of motion can be written as

$$\sum_i \underline{F}_i = m \underline{a}_G$$

$$\sum_i (\underline{M}_G)_i = \sum_i (\underline{r}_i \times \underline{F}_i) = 0$$

or

$$\sum_i \underline{F}_i = m \underline{a}_G$$

$$\sum_i (\underline{M}_P)_i = \sum_i (\underline{p}_i \times \underline{F}_i) = (\underline{r}_{G/P} \times m \underline{a}_G)$$

Fixed Axis Rotation

When a body is undergoing fixed axis rotation as shown in the figure, the equations of motion can be written as

$$\sum_i \underline{F}_i = m \underline{a}_G = m (r \alpha \underline{e}_\theta - r \omega^2 \underline{e}_r) = m (r \ddot{\theta} \underline{e}_\theta - r \dot{\theta}^2 \underline{e}_r)$$

$$\sum_i (\underline{M}_O)_i = \sum_i (\underline{r}_i \times \underline{F}_i) = I_O \alpha = I_O \ddot{\theta} \underline{k}$$

Here I_O represents the *moment of inertia* of the body about a Z axis passing through the *fixed-point* O .

