

Elementary Dynamics

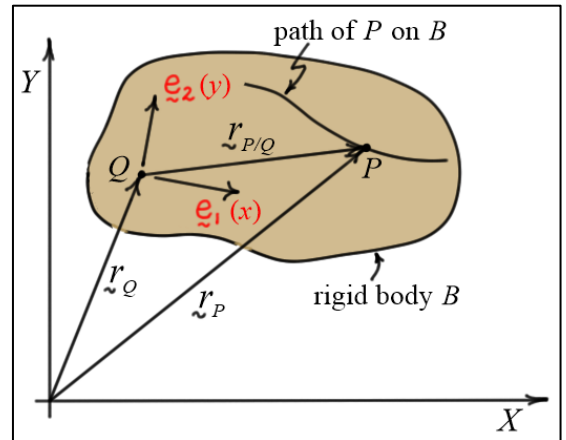
Point Moving on a Rigid Body (Sliding Contact)

The relative motion of *two points fixed on a rigid body* can be calculated using the relative velocity and relative acceleration equations. These equations can be used to analyze many systems such as slider-crank and four-bar mechanisms. However, many systems have *sliding contacts* on *rotating bodies*. These systems cannot be analyzed using the relative velocity or relative acceleration equations. We need a new set of kinematical equations for these systems.

Velocity of a Point Moving on a Rotating Body

Consider the rigid body B shown in the diagram. Point P moves on B , while point Q is fixed on B . The unit vectors \underline{e}_1 and \underline{e}_2 along the x and y directions are fixed in and rotate with B . The position vector of P can be written as

$$\underline{r}_P = \underline{r}_Q + \underline{r}_{P/Q} = \underline{r}_Q + (b_1 \underline{e}_1 + b_2 \underline{e}_2).$$



The velocities of P and Q can be related by differentiating this expression

$$\underline{v}_P = \frac{d\underline{r}_P}{dt} = \frac{d\underline{r}_Q}{dt} + \frac{d}{dt}(b_1 \underline{e}_1 + b_2 \underline{e}_2) = \underline{v}_Q + (\dot{b}_1 \underline{e}_1 + \dot{b}_2 \underline{e}_2) + (b_1 \dot{\underline{e}}_1 + b_2 \dot{\underline{e}}_2)$$

where

$$\dot{b}_1 \underline{e}_1 + \dot{b}_2 \underline{e}_2 \triangleq \underline{v}_{Prel} \quad (\text{the velocity of } P \text{ relative to body } B)$$

$$b_1 \dot{\underline{e}}_1 + b_2 \dot{\underline{e}}_2 = b_1 (\underline{\omega}_B \times \underline{e}_1) + b_2 (\underline{\omega}_B \times \underline{e}_2) = \underline{\omega}_B \times (b_1 \underline{e}_1 + b_2 \underline{e}_2) = \underline{\omega}_B \times \underline{r}_{P/Q}$$

Here, $\underline{\omega}_B$ is the angular velocity of B . Substituting into the expression for \underline{v}_P gives

$$\underline{v}_P = \underline{v}_Q + \underline{v}_{Prel} + \underline{\omega}_B \times \underline{r}_{P/Q}$$

Note this equation is like the relative velocity equation, but it also includes the velocity of P relative to body B . Note also that some texts use a slightly different notation for the velocity of P relative to B . They write $\underline{v}_{Prel} = (\underline{v}_{P/Q})_{xy}$ or $\underline{v}_{Prel} = {}^B \underline{v}_P$.

Acceleration of a Point Moving on a Rotating Body

The accelerations of points P and Q can be related by differentiating again as follows

$$\underline{a}_P = \frac{d\underline{v}_P}{dt} = \frac{d\underline{v}_Q}{dt} + \frac{d\underline{v}_{Prel}}{dt} + \frac{d}{dt}(\underline{\omega}_B \times \underline{r}_{P/Q})$$

where

$$\frac{d\underline{v}_{Prel}}{dt} = \frac{d}{dt}(\dot{b}_1 \underline{e}_1 + \dot{b}_2 \underline{e}_2) = (\ddot{b}_1 \underline{e}_1 + \ddot{b}_2 \underline{e}_2) + (\underline{\omega}_B \times \underline{v}_{Prel}) = \underline{a}_{Prel} + (\underline{\omega}_B \times \underline{v}_{Prel})$$

$$\frac{d}{dt}(\underline{\omega}_B \times \underline{r}_{P/Q}) = (\underline{\alpha}_B \times \underline{r}_{P/Q}) + \left(\underline{\omega}_B \times \frac{d\underline{r}_{P/Q}}{dt} \right) = (\underline{\alpha}_B \times \underline{r}_{P/Q}) + \underline{\omega}_B \times (\underline{v}_{Prel} + (\underline{\omega}_B \times \underline{r}_{P/Q}))$$

Substituting these results into the equation for \underline{a}_P gives

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{Prel} + 2(\underline{\omega}_B \times \underline{v}_{Prel}) + (\underline{\alpha}_B \times \underline{r}_{P/Q}) + \underline{\omega}_B \times (\underline{\omega}_B \times \underline{r}_{P/Q})$$

In two dimensions, this expression can be reduced to

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{Prel} + 2(\underline{\omega}_B \times \underline{v}_{Prel}) + (\underline{\alpha}_B \times \underline{r}_{P/Q}) - \omega_B^2 \underline{r}_{P/Q}$$

Note this equation has two more terms than the relative acceleration equation. It has \underline{a}_{Prel} the acceleration of P *relative* to the **body**, and it also has $2(\underline{\omega}_B \times \underline{v}_{Prel})$ which is called the **Coriolis acceleration**. Note as before that some texts use different notation for the acceleration of P relative to B . They write $\underline{a}_{Prel} = (\underline{a}_{P/Q})_{xy}$ or $\underline{a}_{Prel} = {}^B \underline{a}_P$.

