

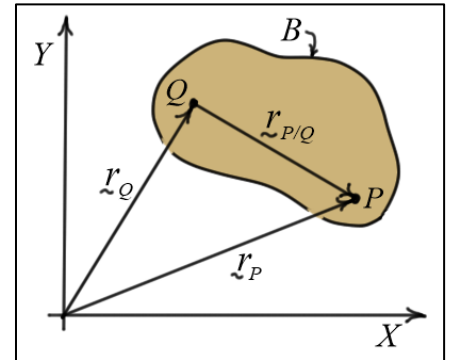
Elementary Dynamics

Relative Acceleration of Two Points Fixed on a Rigid Body

The figure depicts a rigid body B moving in two dimensions. The two points P and Q are *fixed* on B . At any instant of time, the position vector of P can be written as

$$\underline{r}_P = \underline{r}_Q + \underline{r}_{P/Q}$$

Here, $\underline{r}_{P/Q}$ is called the position vector of P relative to Q .



In previous notes, it was shown that this equation could be differentiated to find a relationship between the velocities of P and Q . It was shown

$$\underline{v}_P = \underline{v}_Q + \underline{v}_{P/Q} = \underline{v}_Q + (\underline{\omega} \times \underline{r}_{P/Q}) \quad (\text{relative velocity equation})$$

Here, $\underline{v}_{P/Q}$ is the velocity of P relative to Q which is the *velocity of P* assuming Q is *fixed* and $\underline{\omega}$ is the angular velocity of B . Similarly, the accelerations of points P and Q can be related by *differentiating* the relative velocity equation.

$$\underline{a}_P = \frac{d}{dt}(\underline{v}_Q + (\underline{\omega} \times \underline{r}_{P/Q})) = \frac{d\underline{v}_Q}{dt} + \frac{d}{dt}(\underline{\omega} \times \underline{r}_{P/Q}) = \underline{a}_Q + \left(\frac{d\underline{\omega}}{dt} \times \underline{r}_{P/Q}\right) + \left(\underline{\omega} \times \frac{d\underline{r}_{P/Q}}{dt}\right)$$

or

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{P/Q} = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) + (\underline{\omega} \times \underline{v}_{P/Q}) = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{P/Q})$$

Here, $\underline{\alpha}$ is the angular acceleration of B and, in *two dimensions*, the triple vector product $\underline{\omega} \times (\underline{\omega} \times \underline{r}_{P/Q}) = -\omega^2 \underline{r}_{P/Q}$. Thus, for two-dimensional motion,

$$\underline{a}_P = \underline{a}_Q + \underline{a}_{P/Q} = \underline{a}_Q + (\underline{\alpha} \times \underline{r}_{P/Q}) - \omega^2 \underline{r}_{P/Q}$$

Here, $\underline{a}_{P/Q}$ is the acceleration of P relative to Q which is the *acceleration of P* assuming Q is *fixed*. It can be written in terms of unit vectors \underline{e}_r and \underline{e}_θ as follows.

$$\underline{a}_{P/Q} = (\underline{a}_{P/Q})_t + (\underline{a}_{P/Q})_n = L\alpha \underline{e}_\theta - L\omega^2 \underline{e}_r$$

Here, L is the *distance* between P and Q .

