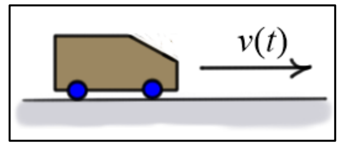
Elementary Engineering Mathematics
Application of Lines in Elementary Dynamics

Example #1

Given: Consider a car moving with ***velocity*** $v(t)$. For a ***constant braking force***, the velocity of the car satisfies the equation:



$$\boxed{v(t) = v_0 + a_0\, t} \qquad (1)$$

Here, v_0 is the velocity of the car at the time the brakes are applied, a_0 is the ***constant acceleration*** of the car until it stops, and t is the time. During a test of the car's braking system, the following data were measured:

Time, t (s)	Velocity, $v(t)$ (ft/s)	Velocity, $v(t)$ (mi/hr)
2.9	74.5	50.8
7.2	30.2	20.6

Find: a) a_0 the constant acceleration of the car; b) v_0 the initial velocity of the car; and c) t^* the time required for the car to stop. Assume a ***constant breaking force*** is applied.

Solution:

Equation (1) is in the slope-intercept form of the equation for a line: $\boxed{y = mx + b}$.

Here, $a_0 = m$ is the slope of the line and $b = v_0$ is the intercept.

a) The slope a_0 can be estimated using the recorded data.

$$\boxed{a_0 = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{30.2 - 74.5}{7.2 - 2.9} = -10.30\ (ft/s^2)}$$

So, we now have: $\boxed{v(t) = -10.30t + v_0}$

b) The intercept v_0 can now be found by using the slope and either of the two data pairs.

$$v(2.9) = 74.5 = -(10.30 \times 2.9) + v_0 \quad \Rightarrow \quad \boxed{v_0 = 74.5 + (10.30 \times 2.9) = 104.4\ (ft/s)}$$

or

$$v(7.2) = 30.2 = -(10.30 \times 7.2) + v_0 \quad \Rightarrow \quad \boxed{v_0 = 30.2 + (10.30 \times 7.2) = 104.4\ (ft/s)}$$

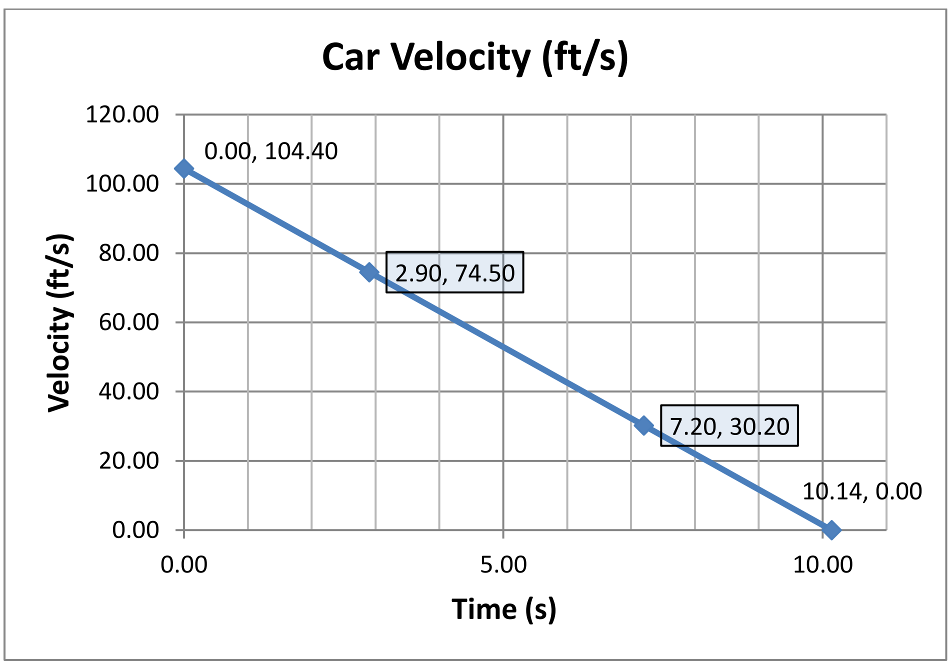
We now have the ***completed equation***: $\boxed{v(t) = -10.30t + 104.4}\ (ft/s)$ (2)

c) Using equation (2) we can find the time t^* required for the car to stop.

$$v(t^*) = 0 = 104.4 - (10.30t^*) \quad \Rightarrow \quad \boxed{t^* = 104.4/10.30 = 10.14\ (s)}$$

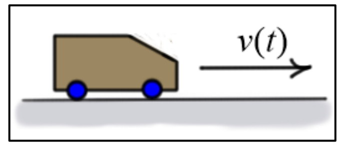
Note:

The ***stopping time*** t^* and the ***initial velocity*** v_0 are the ***x-*** and ***y-intercepts*** of the line.



Example #2

Given: Again, consider a car moving with ***velocity*** $v(t)$. As before, for a ***constant braking force***, the velocity of the car satisfies the equation:



$$\boxed{v(t) = v_0 + a_0\, t} \qquad (3)$$

During a second test of the car's braking system, the following data were measured:

Time, t (s)	Velocity, $v(t)$ (ft/s)	Acceleration, a (ft/s^2)
4.3	59.7	-10.5

Find: a) v_0 the initial velocity of the car; and b) t^* the time required for the car to stop. Assume a constant breaking force is applied.

Solution:

a) To find the initial velocity v_0, we can make use of the ***point-slope form*** of the equation for a line.

$$\frac{(y - y_1)}{(x - x_1)} = m \quad \Rightarrow \quad \big(v(t) - 59.7\big) = -10.5(t - 4.3)$$

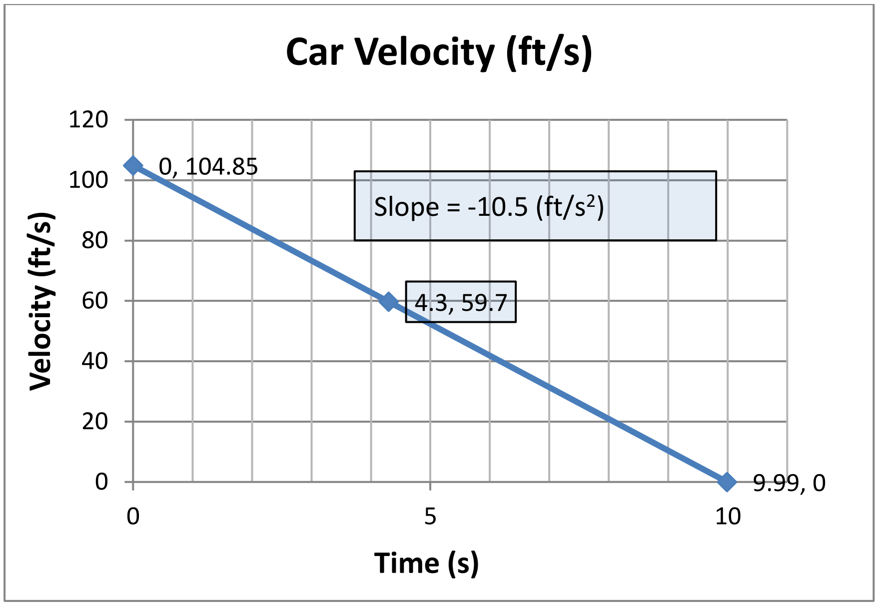
or

$$v(t) = (59.7 + (10.5 \times 4.3)) - 10.5t \quad \Rightarrow \quad \boxed{v(t) = 104.85 - 10.5t}\ (ft/s) \qquad (4)$$

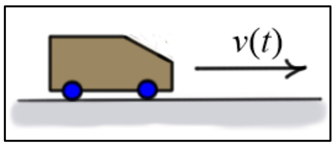
Comparing equations (3) and (4) yields: $\boxed{v_0 = 104.85\,(ft/s) = 71.5\,(mi/hr)}$

b) Equation (4) can also be used to find the time t^*

$$v(t^*) = 0 = 104.85 - (10.5\ t^*) \quad \Rightarrow \quad \boxed{t^* = 104.85 / 10.5 = 9.99\,(s)}$$



<u>Example #3</u>



<u>Given</u>: Now, consider a car that starts at rest, accelerates at a constant rate of $a_0 = 14.8\ (ft/s^2)$ for 6 seconds, and then decelerates at a constant rate $a_1 = -10.5\ (ft/s^2)$ until it stops. Since the car starts from rest, during the ***constant acceleration phase***, the velocity of the car satisfies the equation

$$\boxed{v(t) = a_0\, t} \qquad (5)$$

During the ***constant deceleration phase***, the velocity of the car satisfies the equation

$$\boxed{\big(v(t) - v(t=6)\big) = a_1(t-6)} \qquad \text{(point-slope form)} \qquad (6)$$

<u>Find</u>: a) the equation for $v(t)$ that applies during the deceleration phase, and

b) t^* the time when the car stops.

<u>Solution</u>:

a) Using equation (5), we find the velocity of the car at $t = 6\,(\text{sec})$ to be

$$\boxed{v(t=6) = a_0 t = (14.8)(6) = 88.8\ (ft/s)}$$

Substituting into the point-slope form in equation (6) and reorganizing terms gives

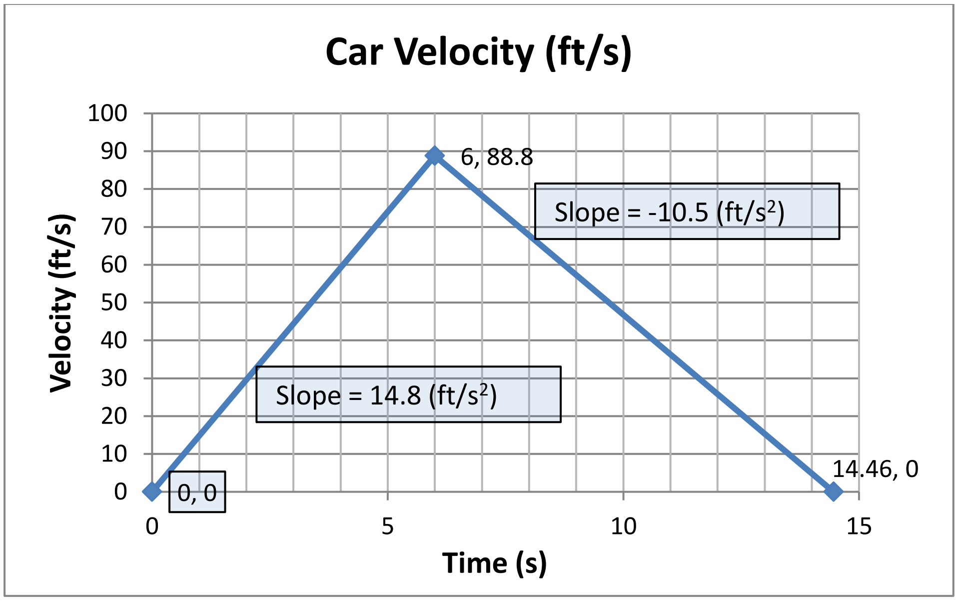
$$v(t) - 88.8 = -10.5(t-6) \quad \Rightarrow \quad v(t) = \left(88.8 + (6\times 10.5)\right) - 10.5t$$

or

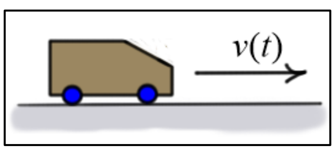
$$\boxed{v(t) = 151.8 - 10.5t} \ (ft/s)$$

b) To find the time t^* when the car stops, set

$$v(t^*) = 0 = 151.8 - (10.5t^*) \quad \Rightarrow \quad \boxed{t^* = 151.8/10.5 = 14.46 \text{ (s)}}$$



Example #4



Given: Now, consider a car that starts at rest, ***accelerates*** at a constant rate of $a_1 (m/s^2)$ for t_1 seconds, and then ***decelerates*** at a constant rate $a_2 (m/s^2)$ until it stops at time t_2. Since the car starts from rest, during the ***constant acceleration phase***, the velocity of the car satisfies the equation

$$\boxed{v(t) = a_1 t} \qquad \left(a_1 > 0\right) \qquad (7)$$

During the ***constant deceleration phase***, the velocity of the car satisfies the equation

$$\boxed{\left(v(t) - v(t_1)\right) = -a_2 (t - t_1)} \qquad \left(a_2 > 0\right) \qquad \text{(point-slope form)} \qquad (8)$$

Find: a) time t_2 in terms of time t_1 and the acceleration and deceleration rates a_1 and a_2, and b) $(t_2 - t_1)/t_1$ the ratio of the time durations of deceleration and acceleration.

Solution:

a) Using the point-slope form in equation (8), and substituting for $v(t_1)$ using equation (7) gives

$$v(t) - v(t_1) = v(t) - a_1 t_1 = -a_2(t - t_1) \quad \Rightarrow \quad \boxed{v(t) = (a_1 + a_2)t_1 - a_2 t} \; (m/s)$$

Now, using the fact that $v(t_2) = 0$, we can solve for the time t_2 as follows

$$v(t_2) = 0 = (a_1 + a_2)t_1 - a_2 t_2 \quad \Rightarrow \quad \boxed{t_2 = \left[\frac{a_1 + a_2}{a_2}\right] t_1} \; (s) \qquad (9)$$

b) Using equation (9), we can solve for the ratio of the time durations of the deceleration and acceleration phases.

$$t_2 - t_1 = \left[\frac{a_1 + a_2}{a_2}\right] t_1 - t_1 = \left[\frac{a_1 + a_2}{a_2} - 1\right] t_1 = \left[\frac{a_1 + \cancel{a_2} - \cancel{a_2}}{a_2}\right] t_1 = \left[\frac{a_1}{a_2}\right] t_1$$

or

$$\boxed{\frac{t_2 - t_1}{t_1} = \frac{a_1}{a_2}}$$