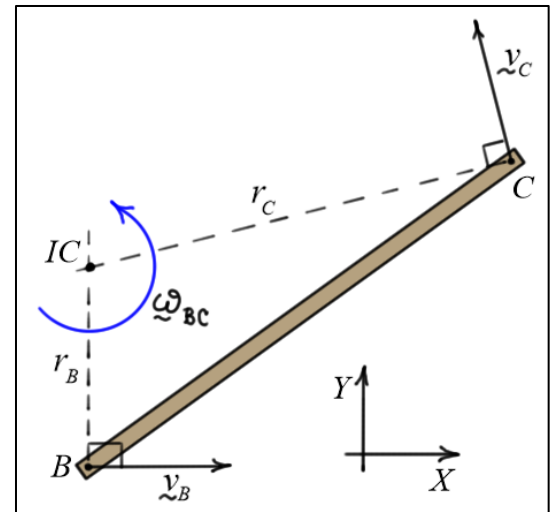


Elementary Engineering Mathematics

Application of Geometry/Trigonometry – Elementary Dynamics

The motion of a rigid body at any instant of time as it moves in the XY plane can be described as *pure rotational motion* about an *instantaneous center (IC)* of *zero velocity*. The location of the *IC* relative to the body (at that instant) can be found by constructing lines *perpendicular* to the velocities of two points on the body. The *intersection* of these two lines (shown as dashed lines in the figure) is the location of the instantaneous center *IC*. Note from instant to instant, the *IC* changes location relative to the body.



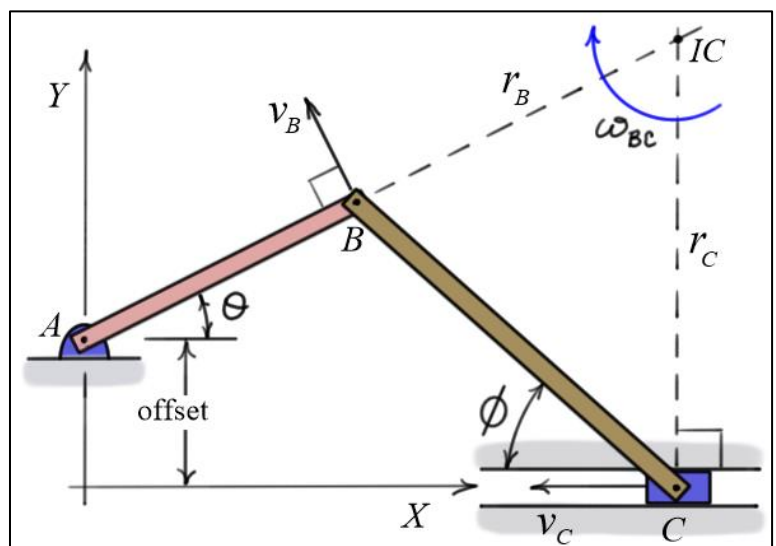
$$\omega_{BC} = \frac{v_B}{r_B} = \frac{v_C}{r_C} \text{ (radians/sec)}$$

The scalar *angular velocity* of the body (i.e. how fast it is rotating about the *IC* in radians/second) is related to the scalar *velocities* of the two points as shown.

Example: Slider-crank mechanism

A slider-crank mechanism with an *offset* is shown in the diagram. Bar AB is the *crank*, piston C is the *slider*, and bar BC is the *connecting rod*. In the position shown, as AB rotates *counterclockwise*, the slider moves to the *left*. The velocity of B is perpendicular to crank AB , and the velocity of C is along the slot (X -axis).

The *instantaneous center (IC)* of BC is found by constructing the dashed lines *perpendicular* to the *velocities* of points B and C . One of these lines is *along* crank AB and the other is *perpendicular* to the slot at C . The *intersection* point of these two lines is the *instantaneous center*.



Problem:

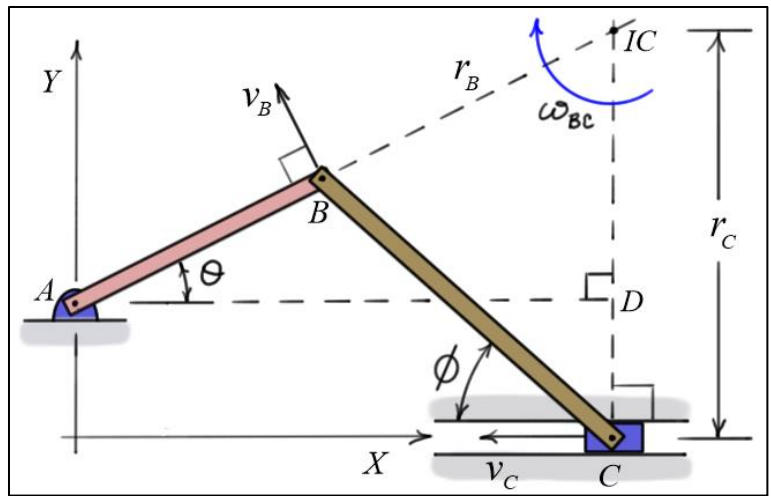
Given: The coordinates of points A, B, and C (in inches) and the velocity of B:

$$A: (0, 3) \quad B: (4, 5) \quad C: (8, 0)$$

$$v_B = 5 \text{ (in/s)} \text{ in direction shown}$$

Find: the location of IC and v_C the velocity of C at this instant.

Solution #1: (using right triangles)



- a) First, construct the dashed lines to the *instantaneous center*. Then *construct* the *right triangle ADIC*, and *calculate* the angle of AB relative to AD.

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) \approx 26.565^\circ$$

- b) *Calculate* the distances r_B and r_C .

$$\tan(\theta) = \frac{r_C - L_{CD}}{L_{AD}} \Rightarrow r_C = L_{CD} + (L_{AD} \tan(\theta)) = 3 + \left(8 \times \frac{2}{4}\right) = 7 \text{ (in)}$$

$$\cos(\theta) = \frac{L_{AD}}{r_B + L_{AB}} \Rightarrow r_B = \left(\frac{L_{AD}}{\cos(\theta)}\right) - L_{AB} \approx \left(\frac{8}{\cos(26.565)}\right) - \sqrt{4^2 + 2^2} \approx 4.47214 \approx 4.47 \text{ (in)}$$

- c) Find the *angular velocity* of BC and the *velocity* of piston C.

$$\omega_{BC} = \frac{v_B}{r_B} \approx \frac{5 \text{ (in/s)}}{4.47214 \text{ (in)}} \approx 1.118 \approx 1.12 \text{ (rad/sec)} \text{ (angular motion is } \mathbf{clockwise})$$

$$\frac{v_B}{r_B} = \frac{v_C}{r_C} \Rightarrow v_C = \left(\frac{r_C}{r_B}\right) v_B \approx \left(\frac{7}{4.47214}\right) 5 \approx 7.82624 \approx 7.83 \text{ (in/s)}$$

Note: When analyzing slider-crank mechanisms, we have the *advantage* of being able to use *right triangles*; however, for more complex mechanisms (such as *four-bar* mechanisms), we will often need a more *general approach* using *non-right triangles*.

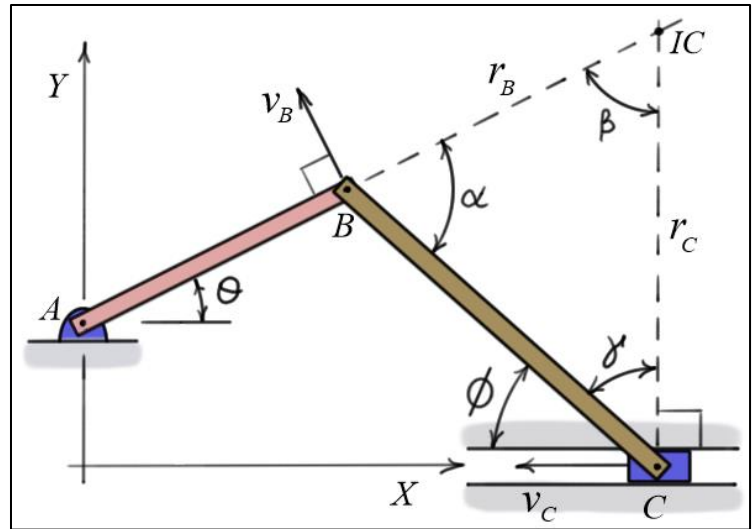
Solution #2: (using non-right triangles)

- a) First, calculate the angles of AB and BC relative to the X -axis.

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) \approx 26.565^\circ$$

$$\phi = \tan^{-1}\left(\frac{5}{4}\right) \approx 51.34^\circ$$

- b) Construct the dashed lines to the *instantaneous center*. In the newly constructed triangle $BCIC$, define the *unknown angles* α , β , and γ .



- c) *Calculate* the *unknown angles* using the concepts from geometry.

$$\alpha = \theta + \phi \approx 77.905^\circ \quad \beta = 90 - \theta \approx 63.435^\circ \quad \gamma = 90 - \phi \approx 38.66^\circ \quad \dots \text{why?}$$

- d) Now use the *law of sines* to find the distances r_B and r_C .

$$\frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{r_B} \Rightarrow \frac{\sin(63.435)}{\sqrt{4^2 + 5^2}} = \frac{\sin(38.66)}{r_B} \Rightarrow r_B = \frac{\sin(38.66)}{\sin(63.435)} \sqrt{4^2 + 5^2}$$

$$\Rightarrow r_B \approx 4.47215 \approx 4.47 \text{ (inches)}$$

$$\frac{\sin(\beta)}{b} = \frac{\sin(\alpha)}{r_C} \Rightarrow \frac{\sin(63.435)}{\sqrt{4^2 + 5^2}} = \frac{\sin(77.905)}{r_C} \Rightarrow r_C = \frac{\sin(77.905)}{\sin(63.435)} \sqrt{4^2 + 5^2}$$

$$\Rightarrow r_C \approx 7.0 \text{ (inches)}$$

- e) Find the *angular velocity* of BC and the *velocity* of piston C as shown above.