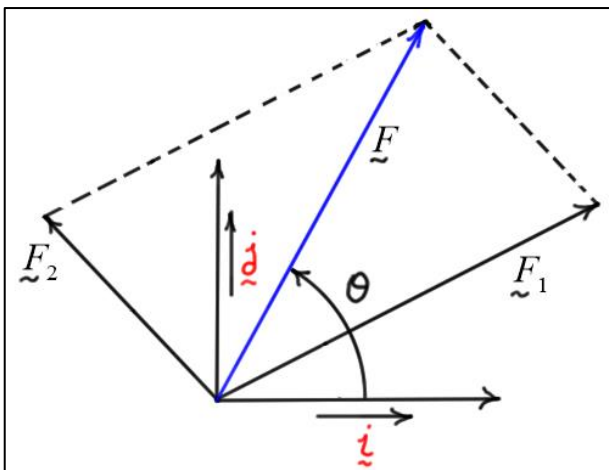
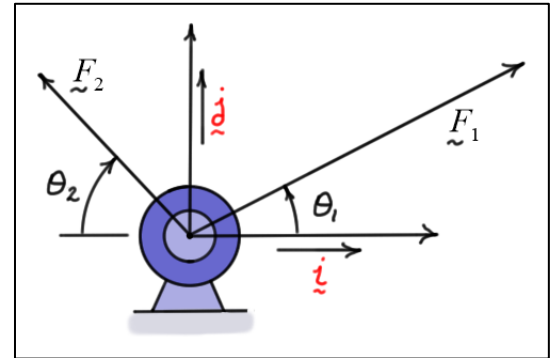


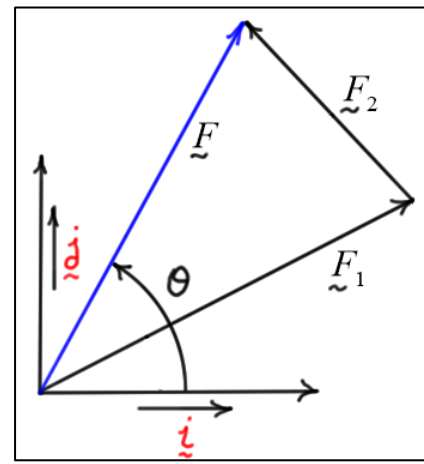
Elementary Statics

Parallelogram Law of Vector Addition

- The most basic way to add two vectors is a *geometric approach* called the *parallelogram law* of *vector addition*.
- The *sum* of the two vectors is called the *resultant*.
- Consider the two forces \vec{F}_1 and \vec{F}_2 acting on the support as shown. The total force acting on the support is found by *adding* the two vectors.
- By placing the tails of the vectors at the same point, we can *construct* the resultant $\vec{F} \triangleq \vec{F}_1 + \vec{F}_2$ by forming a *parallelogram* as shown in the diagram below on the left. By placing the tail of the second vector at the head of the first, we can construct the resultant by forming a *triangle* as shown in the diagram on the right.



Parallelogram formed by \vec{F}_1 and \vec{F}_2



Triangle formed by \vec{F}_1 and \vec{F}_2

- In general, the triangle formed by \vec{F}_1 , \vec{F}_2 , and the resultant \vec{F} is a *non-right triangle*. The lengths and angles within this triangle can be related using the *law of cosines* and the *law of sines*.

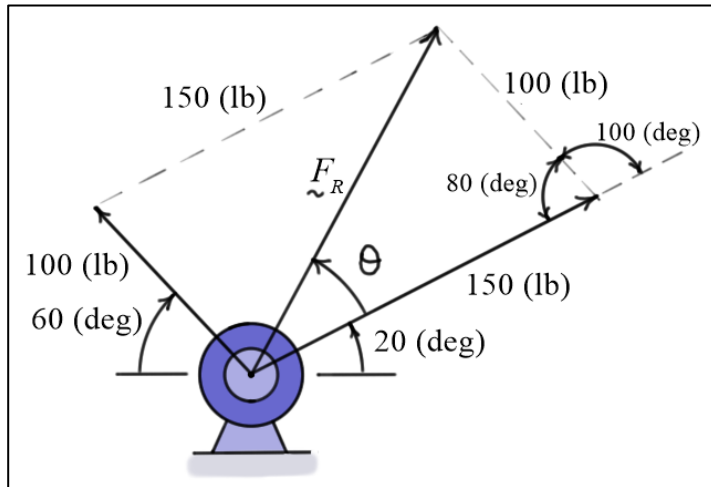
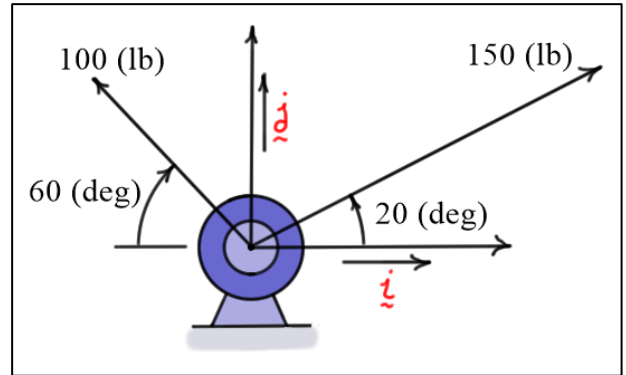
Example #1:

Given: Two forces in the diagram

Find: \vec{F}_R the resultant force

Solution:

Geometric construction:



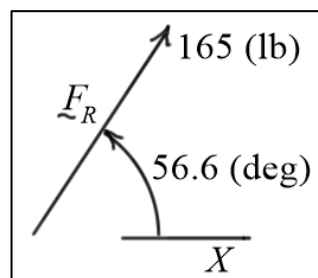
Applying the law of cosines and law of sines to the diagram above gives.

Law of Cosines: $F_R^2 = 150^2 + 100^2 - 2(100)(150)\cos(80) \approx 27290.6$

$\Rightarrow F_R \approx 165.199 \approx 165 \text{ (lb)}$

Law of Sines: $\frac{\sin(\theta)}{100} = \frac{\sin(80)}{F_R} \Rightarrow \theta = \sin^{-1}\left(\frac{100\sin(80)}{F_R}\right) \approx 36.5937 \approx 36.6 \text{ (deg)}$

Result: \vec{F}_R is a 165 (lb) force acting at 56.6 (deg) relative to the X direction.



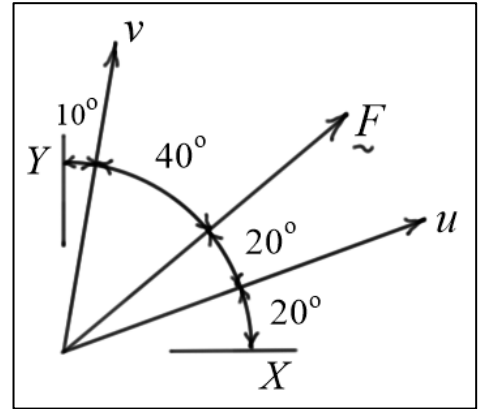
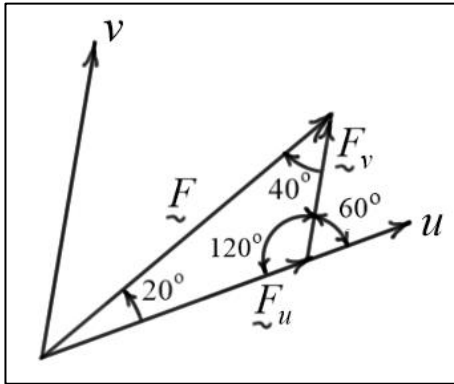
Example #2:

Given: $|\vec{F}| = 200$ (lb) is oriented as shown in the diagram

Find: F_u and F_v the components of \vec{F} along the u and v directions

Solution:

Geometric construction:



As drawn, $\vec{F} = \vec{F}_u + \vec{F}_v$. Using the law of sines, write

$$\frac{F_u}{\sin(40)} = \frac{F_v}{\sin(20)} = \frac{F}{\sin(120)}$$

$$\Rightarrow F_u = \left(\frac{\sin(40)}{\sin(120)} \right) F = \left(\frac{\sin(40)}{\sin(120)} \right) 200 \approx 148.445 \approx 148 \text{ (lb)}$$

$$\Rightarrow F_v = \left(\frac{\sin(20)}{\sin(120)} \right) F = \left(\frac{\sin(20)}{\sin(120)} \right) 200 \approx 78.9862 \approx 79.0 \text{ (lb)}$$