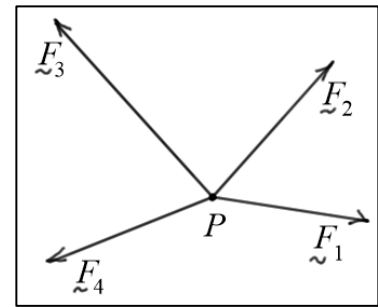


Elementary Statics

Equilibrium of a Particle

- If a particle P does *not move*, it is said to be in *static equilibrium*.
- To study the *forces* necessary for equilibrium, we can isolate P as a *free body*, and *identify* all the *forces acting on it*. This is called a *free body diagram*, and it is necessary to be clear about *how* the forces act on P .



Free Body Diagram

- For P to be in *static equilibrium*, the *sum* of all forces (or resultant force) acting on it must be *zero*. The diagram shows a particle with four forces acting on it. For a particle with N forces acting on it.

$$\sum_{i=1}^N \vec{F}_i = \vec{0}$$

This means that the *sum* of the forces in *all directions* must be *zero*.

- In two dimensions (2D), this *vector equation* represents *two scalar equations*, and in three dimensions (3D), it represents *three scalar equations*.

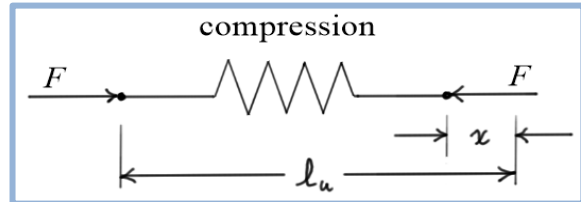
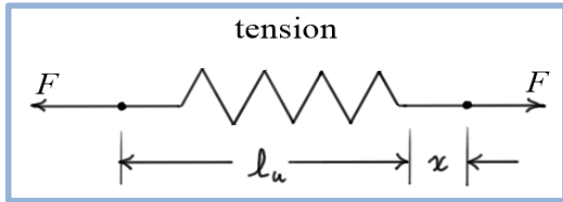
$$\begin{cases} \sum_i (F_x)_i = 0 \\ \sum_i (F_y)_i = 0 \end{cases} \quad (2D)$$

$$\begin{cases} \sum_i (F_x)_i = 0 \\ \sum_i (F_y)_i = 0 \\ \sum_i (F_z)_i = 0 \end{cases} \quad (3D)$$

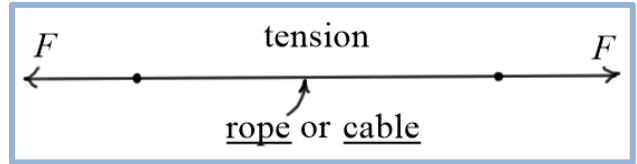
- The *free body diagram* often contains *unknown forces* we are trying to find. In 2D problems, we can find up to *two unknowns*, and in 3D problems, we can find up to *three unknowns*.

Assumptions for Some Typical Structural Components

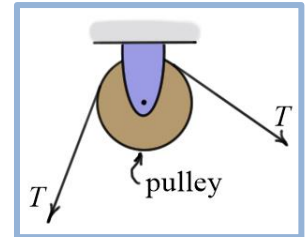
- *Springs* are assumed to generate forces *proportional* to the *elongation* or *compression* of the spring from its *natural* (or *unstretched*) length, ℓ_u . When stretched, the spring is said to be in “*tension*” and when it is compressed, it is said to be in “*compression*.” The force required to hold the spring in tension or compression is $F = kx$.



- As a first approximation, **cables** or **ropes** are assumed to form **straight lines** with **no sag**. They can only act in **tension**. Forces are transmitted along the line.



- Forces are often **redirected** using **pulleys**. As a first approximation, assume pulleys are **massless** and **frictionless**, so the tension in the rope or cable is the same on both sides of the pulley.



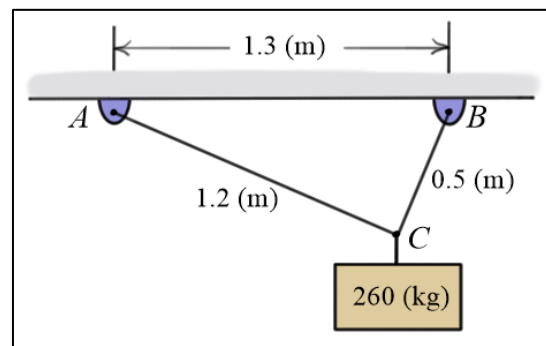
Example #1:

Given: Two cables support the 260 (kg) load.

Find: Tensions in the two cables.

Solution:

Geometry: Using the law of cosines, write



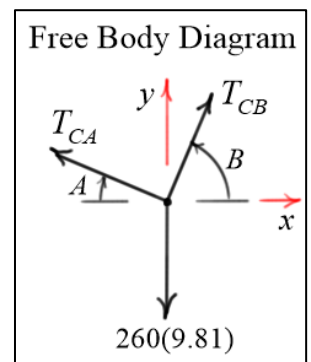
$$0.5^2 = 1.2^2 + 1.3^2 - 2(1.2)(1.3)\cos(A) \Rightarrow A = \cos^{-1}\left(\frac{1.2^2 + 1.3^2 - 0.5^2}{2(1.2)(1.3)}\right) \approx 22.6199 \text{ (deg)}$$

$$1.2^2 = 0.5^2 + 1.3^2 - 2(0.5)(1.3)\cos(B) \Rightarrow B = \cos^{-1}\left(\frac{0.5^2 + 1.3^2 - 1.2^2}{2(0.5)(1.3)}\right) \approx 67.3801 \text{ (deg)}$$

Equations of equilibrium:

$$\sum F_x = -\cos(A)T_{CA} + \cos(B)T_{CB} = 0$$

$$\sum F_y = \sin(A)T_{CA} + \sin(B)T_{CB} - 260(9.81) = 0$$



Simultaneous equations:

$$\begin{bmatrix} -\cos(A) & \cos(B) \\ \sin(A) & \sin(B) \end{bmatrix} \begin{Bmatrix} T_{CA} \\ T_{CB} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 260(9.81) \end{Bmatrix} \Rightarrow \begin{Bmatrix} T_{CA} \\ T_{CB} \end{Bmatrix} \approx \begin{Bmatrix} 981 \text{ (N)} \\ 2350 \text{ (N)} \end{Bmatrix}$$

Example #2:

Given:

Three cables support weight W as shown.

Tension in cable DB is measured to be 975 (lb).

Find:

Tensions in cables DA and DC and weight W .

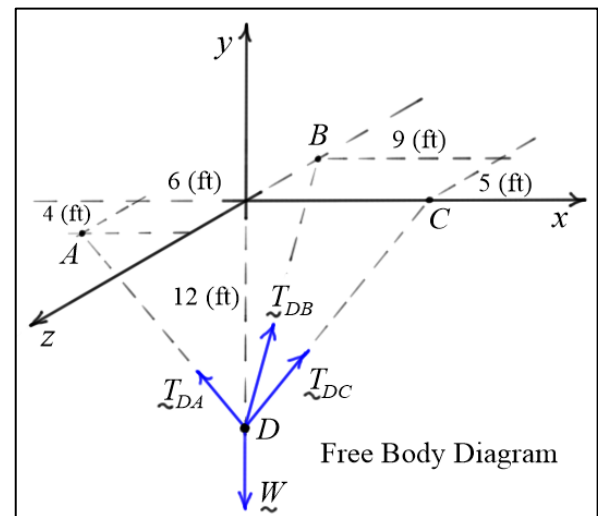
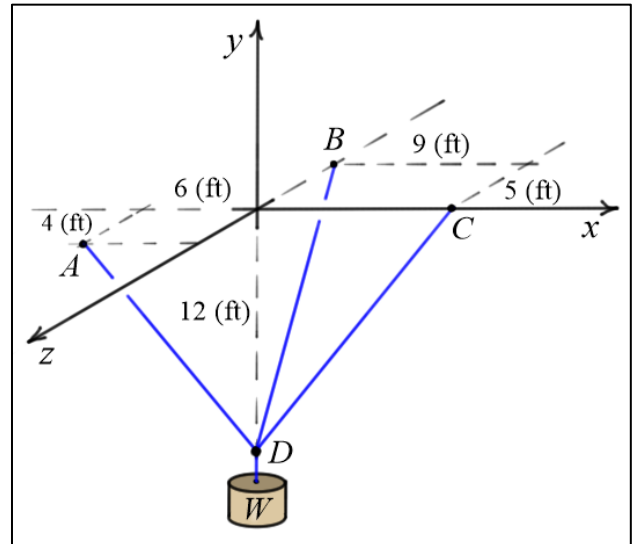
Solution:

$$\underline{\underline{\vec{W} = -W \underline{j}}}$$

$$\begin{aligned} \underline{T}_{DB} &= 975 \underline{n}_{DB} = 975(12 \underline{j} - 5 \underline{k}) / \sqrt{12^2 + 5^2} \\ &= 975(12 \underline{j} - 5 \underline{k}) / 13 \Rightarrow \underline{\underline{T}_{DB} = 900 \underline{j} - 375 \underline{k}} \end{aligned}$$

$$\begin{aligned} \underline{T}_{DC} &= T_{DC} \underline{n}_{DC} = T_{DC}(9 \underline{i} + 12 \underline{j}) / \sqrt{9^2 + 12^2} \\ &= T_{DC}(9 \underline{i} + 12 \underline{j}) / 15 \Rightarrow \underline{\underline{T}_{DC} = T_{DC}(3 \underline{i} + 4 \underline{j}) / 5}} \end{aligned}$$

$$\begin{aligned} \underline{T}_{DA} &= T_{DA} \underline{n}_{DA} = T_{DA}(-6 \underline{i} + 12 \underline{j} + 4 \underline{k}) / \sqrt{6^2 + 12^2 + 4^2} \\ &= T_{DA}(-6 \underline{i} + 12 \underline{j} + 4 \underline{k}) / 14 \\ &\Rightarrow \underline{\underline{T}_{DA} = T_{DA}(-3 \underline{i} + 6 \underline{j} + 2 \underline{k}) / 7}} \end{aligned}$$



Equations of Equilibrium:

$$\boxed{\sum F_x = -\frac{3}{7}T_{DA} + \frac{3}{5}T_{DC} = 0} \quad \boxed{\sum F_y = \frac{6}{7}T_{DA} + \frac{4}{5}T_{DC} + 900 - W = 0} \quad \boxed{\sum F_z = \frac{2}{7}T_{DA} - 375 = 0}$$

Solving:

The third equation gives

$$\underline{\underline{T_{DA} = \frac{7}{2}(375) = 1312.5 \approx 1310 \text{ (lb)}}}$$

Substituting this result into the first equation gives

$$\underline{\underline{T_{DC} = \left(\frac{5}{3}\right)\left(\frac{3}{7}\right)T_{DA} = 937.5 \approx 938 \text{ (lb)}}}$$

Substituting these two results into the second equation gives

$$\underline{\underline{W = \frac{6}{7}T_{DA} + \frac{4}{5}T_{DC} + 900 = 2775 \approx 2780 \text{ (lb)}}}$$