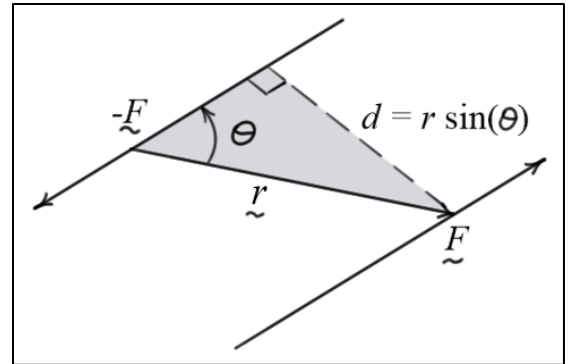


Elementary Statics

Force Couples

- **Definition:** A *force couple* is a pair of *equal and opposite forces* whose lines of action are separated by some distance, d .
- A force couple is simply referred to as a *couple*.
- As the forces are *equal and opposite*, the *resultant* of the two forces is *zero*.



- It can be shown that the *sum* of the *moments* of the two forces about *any point* is

$$\underline{M} = \underline{r} \times \underline{F}$$

Here, \underline{r} is the *position vector* from *any point* on the *line of action* of one of the forces to *any point* on the *line of action* of the other force.

- The *magnitude* of the moment (or torque) is $M = |\underline{M}| = F r \sin(\theta) = F d$.
- As before, the *direction* of the moment is given by the *right-hand rule* and is *perpendicular* to the *plane* formed by the two forces.
- The *moment* of the *couple* is often referred to as a *couple moment*.
- Because the *moment* of a couple is the *same* about all points, it is a *free vector*.

Resultant of a Set of Couples

- The *resultant force* of a set of couples is *zero*.
- The *resultant moment* of a set of couples is the *vector sum* of the *couple moments*.

Example #1:

Given: $|F| = F = 80$ (N)

Find: M_C the moment (or torque) of the couple

Solution:

$$\underline{M}_C = \underline{r}_{B/A} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = (0.5(80))\underline{i} - (0.1(80))\underline{j} \Rightarrow \underline{M}_C = 40\underline{i} - 8\underline{j} \text{ (N-m)}$$

$$|\underline{M}_C| = \sqrt{40^2 + 8^2} \approx 40.8 \text{ (N-m)}$$

Check: Recall the moment of a couple is the same about all points. So, we can also write

$$\begin{aligned} \underline{M}_C &= (\underline{r}_{B/O} \times \underline{F}) + (\underline{r}_{A/O} \times -\underline{F}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.3 & 0.8 & 0 \\ 0 & 0 & 80 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.2 & 0.3 & 0 \\ 0 & 0 & -80 \end{vmatrix} \\ &= (0.8(80) - 0.3(80))\underline{i} - (0.3(80) - 0.2(80))\underline{j} \\ &\Rightarrow \underline{M}_C = 40\underline{i} - 8\underline{j} \text{ (N-m)} \text{ (same result)} \end{aligned}$$

Example #2:

Given: $|M_C| = 50$ (N-m)

Find: F

Solution:

$$\underline{M}_C = \underline{r}_{B/A} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = (0.5F)\underline{i} - (0.1F)\underline{j}$$

$$|\underline{M}_C| = \sqrt{0.5^2 F^2 + 0.1^2 F^2} = F\sqrt{0.5^2 + 0.1^2} \Rightarrow F = \frac{50}{\sqrt{0.5^2 + 0.1^2}} \approx 98.1 \text{ (N)}$$

