Elementary Statics
Equivalent Force Systems

- ***Two systems*** of forces and couples are called ***equivalent force systems*** if the two systems have the ***same resultant force*** and the ***same resultant moment*** about ***any point***.
- This idea can be used to ***reduce*** a ***system*** of forces and couples to a ***single force-couple system*** acting at some point *O*. The ***single force*** $\left(\underset{\sim}{F}\right)$ is the ***resultant force*** of the system, and the ***single couple-moment*** $\left(\underset{\sim}{M}_O\right)$ is the ***sum*** of the ***moments*** of ***all the forces*** about *O* ***plus*** the ***sum*** of ***all*** the ***couple-moments***.

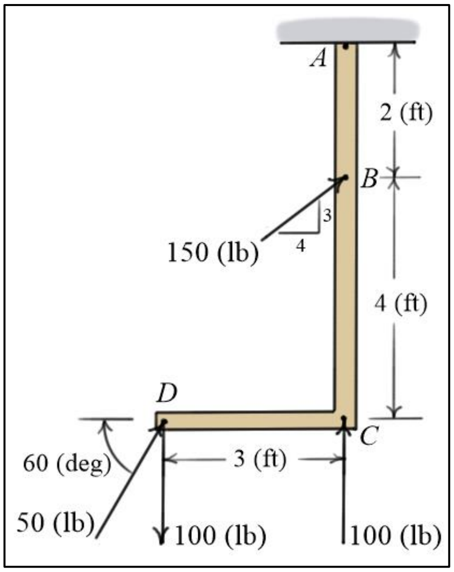
$$\boxed{\begin{aligned}&\underset{\sim}{F} = \underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i\\ &\underset{\sim}{M}_O = \left(\underset{\sim}{M}_O\right)_R = \sum_{\text{forces } (i)} \left(\underset{\sim}{r}_i \times \underset{\sim}{F}_i\right) + \sum_{\text{couples } (i)} \left(\underset{\sim}{M}_C\right)_i\end{aligned}}$$

- If we are studying the ***external forces*** acting on a body, we ***can*** use ***equivalent force systems*** to ***simplify*** our work.
- If we are studying the ***internal forces*** within a body, we need to be careful when using equivalent force systems. The equivalent force systems must produce the same internal effects.

<u>Example #1</u>:

Given: system of forces shown

Find: Equivalent force-couple system at *A*



Solution:

Force acting at *A*:

$$\underset{\sim}{F}_R = \underset{\sim}{F}_B + \underset{\sim}{F}_D = 150\left(\tfrac{4}{5}\underset{\sim}{i} + \tfrac{3}{5}\underset{\sim}{j}\right) + 50\left(\cos(60)\underset{\sim}{i} + \sin(60)\underset{\sim}{j}\right)$$

$$= \left(120 + 50(\tfrac{1}{2})\right)\underset{\sim}{i} + \left(90 + 50(\tfrac{\sqrt{3}}{2})\right)\underset{\sim}{j}$$

$$\Rightarrow \boxed{\underset{\sim}{F}_R \approx 145\underset{\sim}{i} + 133\underset{\sim}{j}}$$

Corresponding couple-moment:

$$\underset{\sim}{M}_C = \sum \underset{\sim}{M}_A = \left[2\left(\tfrac{4}{5}(150)\right) + 6\left(50\cos(60)\right) - 3\left(50\sin(60)\right) + 3(100)\right]\underset{\sim}{k} \text{ (ft-lb)}$$

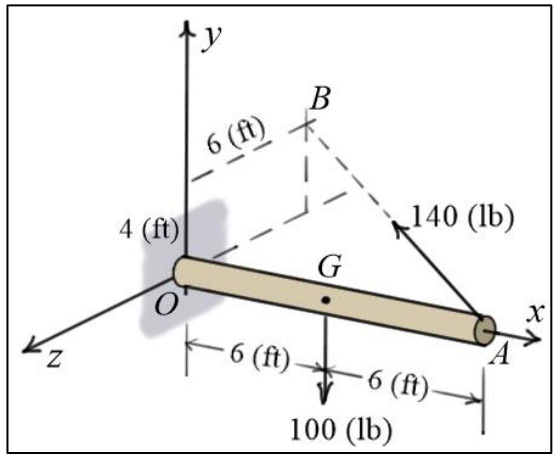
$$\approx \left[240 + 150 - 129.904 + 300\right]\underset{\sim}{k} \text{ (ft-lb)}$$

$$\Rightarrow \boxed{\underset{\sim}{M}_C \approx 560\underset{\sim}{k} \text{ (ft-lb)}}$$

Example #2:

Given: The 12-foot beam is fixed into the wall at O and is subjected to its weight of 100-pound and the 140-pound cable force at A.

Find: Equivalent force and couple-moment load at O.



Solution:

Force acting at O:

$$\underset{\sim}{F}_R = 140\left[\left(-12\,\underset{\sim}{i} + 4\,\underset{\sim}{j} - 6\,\underset{\sim}{k}\right)\Big/\sqrt{12^2 + 4^2 + 6^2}\,\right] - 100\,\underset{\sim}{j}$$

$$= 140\left[\left(-12\,\underset{\sim}{i} + 4\,\underset{\sim}{j} - 6\,\underset{\sim}{k}\right)\Big/14\right] - 100\,\underset{\sim}{j} = \left(-120\,\underset{\sim}{i} + 40\,\underset{\sim}{j} - 60\,\underset{\sim}{k}\right) - 100\,\underset{\sim}{j}$$

$$\Rightarrow \boxed{\underset{\sim}{F}_R = -120\,\underset{\sim}{i} - 60\,\underset{\sim}{j} - 60\,\underset{\sim}{k}} \text{ (lb)}$$

Corresponding couple-moment:

$$\underset{\sim}{M}_C = \sum \underset{\sim}{M}_O = \left[\left(6\,\underset{\sim}{i}\right)\times\left(-100\,\underset{\sim}{j}\right)\right] + \left[\left(12\,\underset{\sim}{i}\right)\times\left(-120\,\underset{\sim}{i} + 40\,\underset{\sim}{j} - 60\,\underset{\sim}{k}\right)\right]$$

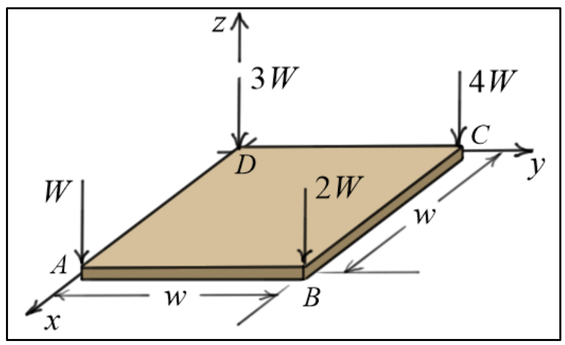
$$= \left[-600\,\underset{\sim}{k}\right] + \left[12(40)\,\underset{\sim}{k} + 12(60)\,\underset{\sim}{j}\right]$$

$$\Rightarrow \boxed{\underset{\sim}{M}_C = 720\,\underset{\sim}{j} - 120\,\underset{\sim}{k}} \text{ (ft-lb)}$$

Example #3:

Given: The square plate supports four loads as shown.

Find: The resultant load and where it acts on the plate.



Solution:

Resultant Force:

$$\boxed{\underset{\sim}{F}_R = -(1+2+4+3)W\,\underset{\sim}{k} = -10W\,\underset{\sim}{k}}$$

Resultant Moment about D:

$$\underset{\sim}{M}_D = \left[w\,\underset{\sim}{i} \times -W\,\underset{\sim}{k}\right] + \left[\left(w\,\underset{\sim}{i} + w\,\underset{\sim}{j}\right)\times -2W\,\underset{\sim}{k}\right] + \left[w\,\underset{\sim}{j} \times -4W\,\underset{\sim}{k}\right]$$

$$= \left[wW\,\underset{\sim}{j}\right] + \left[2wW\,\underset{\sim}{j} - 2wW\,\underset{\sim}{i}\right] + \left[-4wW\,\underset{\sim}{i}\right] \qquad \Rightarrow \boxed{\underset{\sim}{M}_D = -6wW\,\underset{\sim}{i} + 3wW\,\underset{\sim}{j}}$$

If $\underset{\sim}{F}_R$ is located at the coordinates $(\hat{x}, \hat{y})$ on the plate relative to D, its moment about D can be calculated as follows.

$$\boxed{\underset{\sim}{M}_D = \left(\hat{x}\,\underset{\sim}{i} + \hat{y}\,\underset{\sim}{j}\right)\times\left(-10W\,\underset{\sim}{k}\right) = -10\hat{y}W\,\underset{\sim}{i} + 10\hat{x}W\,\underset{\sim}{j}}$$

Equating the two expressions $\underset{\sim}{M}_D$ gives

$$-10\hat{y} = -6w \qquad \Rightarrow \boxed{\hat{y} = 0.6w}$$

$$10\hat{x} = 3w \qquad \Rightarrow \boxed{\hat{x} = 0.3w}$$