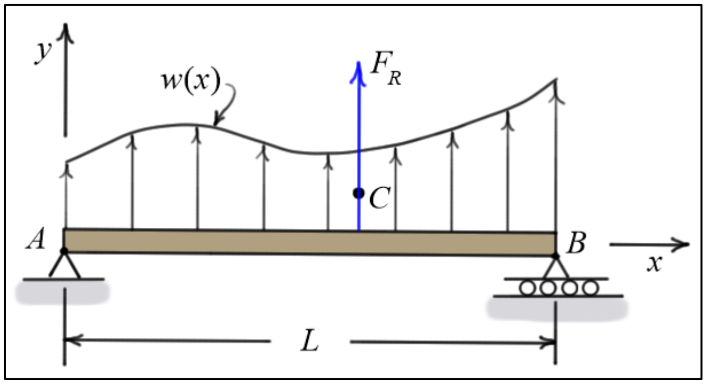
Elementary Statics
Equivalent Force Systems for Distributed Loads



- It is common for ***structural*** members to experience loads that are ***distributed*** along parts or all their length.
- The diagram shows a ***simply supported beam*** with ***distributed load*** $w(x)$. The units of $w(x)$ are pounds per foot (***lb/ft***) or Newtons per meter (***N/m***).
- To find the ***external forces*** acting on the beam shown at its supports at *A* and *B*, it is helpful to ***replace*** the distributed load by an ***equivalent force system***.
- In this case, the ***equivalent force system*** is simply a ***single resultant force*** F_R acting at the ***centroid*** of the ***area*** under the load diagram.

$$\boxed{F_R = \sum F = \int_0^L w(x)\,dx}$$

- Because the resultant force can be moved along its line of action, we only need to find $\bar{x}$ the ***x-coordinate*** of the centroid of the load area**.**

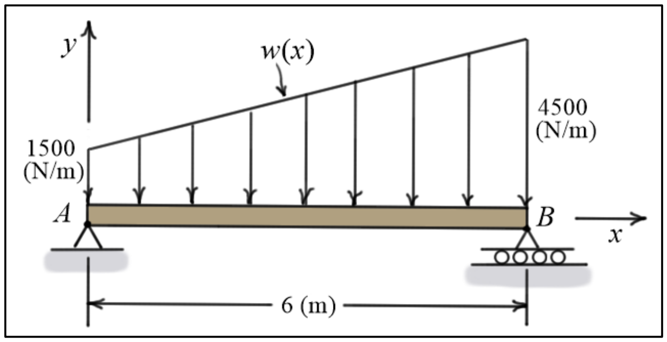
$$\boxed{\bar{x} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx} = \frac{1}{F_R}\int_0^L x\,w(x)\,dx} \quad \text{or} \quad \boxed{\bar{x}\,F_R = \int_0^L x\,w(x)\,dx}$$

- Here, the term $\bar{x}\,F_R$ represents the ***moment*** of the ***resultant force*** about point *A*, and the term $\int_0^L x\,w(x)\,dx$ represents the ***sum*** of the ***moments*** of the ***distributed load*** about point *A*.
- As mentioned in previous notes, if we are studying the ***internal forces*** within a body, we ***cannot*** use equivalent force systems to represent the external loads. We ***must use*** the forces and couple moments ***as applied***.

Example: (using composite shapes)

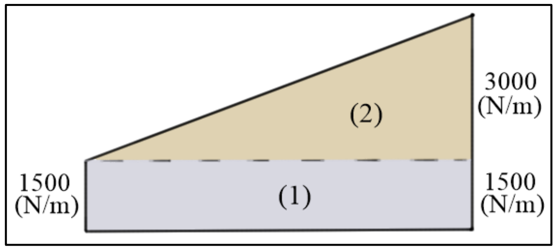
Given: Beam loaded as shown

Find: Resultant F_R and its location relative to end A



Solution:

For the purpose of finding the resultant and its location, the applied load can be thought of having two parts, a constant distributed load of 1500 (N/m) and a triangular distributed load which increases from zero at A to 3000 (N/m) at B.



Constant distributed load:

$$\boxed{\underset{\sim}{F}_1 = -6(1500)\,\underset{\sim}{j} = -9000\,\underset{\sim}{j} = -9\,\underset{\sim}{j}\ \text{(kN)}}$$ acting at $\boxed{\bar{x}_1 = 3\ \text{(m)}}$ the midpoint of the beam

Triangular distributed load:

$$\boxed{\underset{\sim}{F}_2 = -\tfrac{1}{2}(6)(3000)\,j = -9000\,\underset{\sim}{j} = -9\,\underset{\sim}{j}\ \text{(kN)}}$$ acting at $\boxed{\bar{x}_2 = \tfrac{2}{3}(6) = 4\ \text{(m)}}$.

Total load:

$$\boxed{\underset{\sim}{F}_R = \sum_{i=1}^{2} \underset{\sim}{F}_i = -18000\,\underset{\sim}{j}\ \text{(N)} = -18\,\underset{\sim}{j}\ \text{(kN)}}$$

$$\boxed{\bar{x} = \tfrac{1}{F_R}\left(F_1\,\bar{x}_1 + F_2\,\bar{x}_2\right) = \tfrac{1}{18}\left(9(3) + 9(4)\right) = \tfrac{7}{2} = 3.5\ \text{(m)}}$$