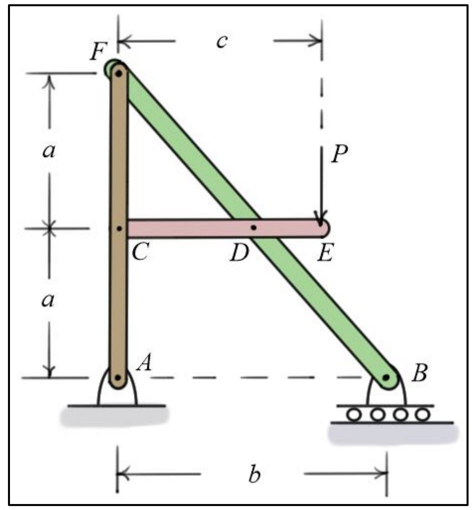
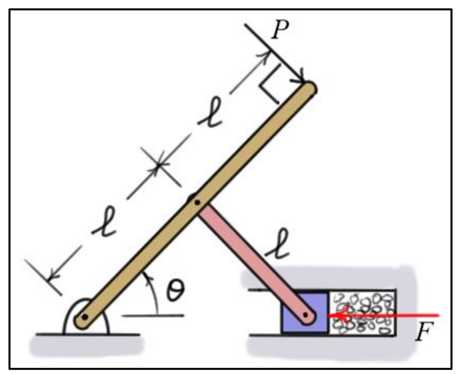
Elementary Statics
Frames and Machines

Frames



- Frames are ***load supporting structures*** whose members are usually ***connected*** with ***smooth pins***.
- The members can be ***two-force*** or ***multi-force*** members.
- External Support Reaction Forces:
  - The ***external support reaction forces*** can often be found by writing the ***equilibrium equations*** for the ***entire frame***. In this case, a ***free body diagram*** is drawn of the ***entire frame***.
- Individual Member Forces:
  - The ***individual member forces*** are found by writing the ***equilibrium equations*** for ***each member*** of the frame.
  - ***Free body diagrams*** are drawn for ***each member***, making sure to obey ***Newton's third law*** of ***action*** and ***reaction*** between ***adjoining members***.

Machines

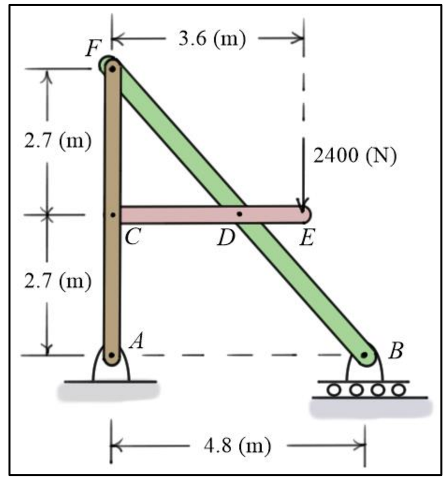


- Machines are used to ***transmit applied forces*** for some useful purpose.
- Members of machines are usually ***connected*** with ***smooth pins***.
- Members can be ***two-force*** or ***multi-force*** members.
- External Support Reaction Forces (if any):
  - The ***external support reaction forces*** can often be found by writing the ***equilibrium equations*** for the entire machine.
  - In this case, a ***free body diagram*** is drawn of the ***entire machine***.
- Individual Member Forces:
  - The ***individual member forces*** are found by writing the ***equilibrium equations*** for ***each member*** of the machine.
  - ***Free body diagrams*** are drawn for ***each member***, making sure to obey ***Newton's third law*** of action and reaction between ***adjoining members***.

Example #1:

Given: Frame loaded as shown.

Find: All external support and member forces



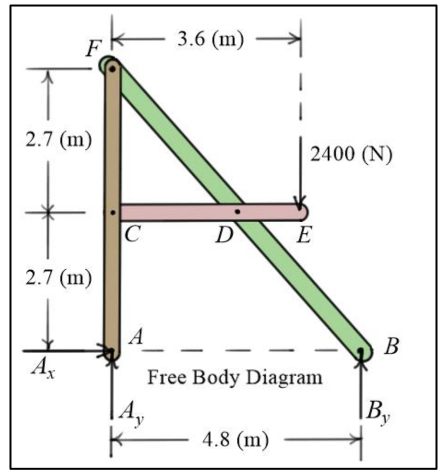
Solution: (neglecting member weights)

External Support Forces: (using free body diagram of entire frame)

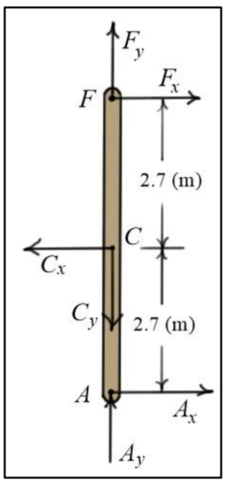
$$\circlearrowleft\!+\ \sum M_A = 4.8B_y - 3.6(2400) = 0 \quad \Rightarrow \boxed{B_y = 1800\ (\text{N})}$$

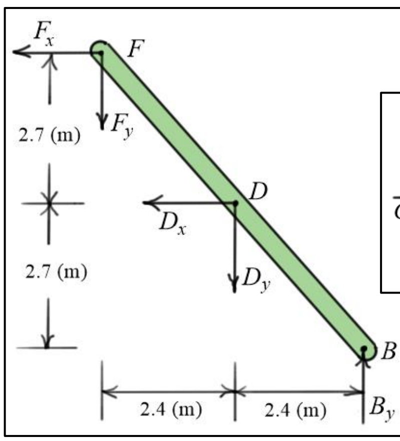
$$\circlearrowleft\!+\ \sum M_B = -4.8A_y + (4.8-3.6)2400 = 0 \quad \boxed{A_y = 600\ (\text{N})}$$

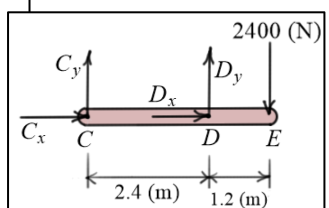
$$\boxed{\sum F_x = A_x = 0}$$



Member Free Body Diagrams:

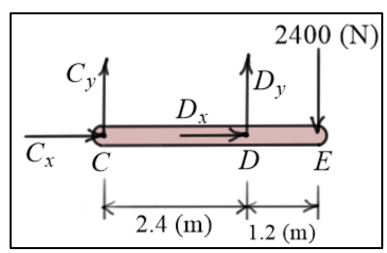






Note: Consistent with Newton's third law, the pin forces on any two adjoining members are equal and opposite on the free body diagrams.

Member *CDE*:

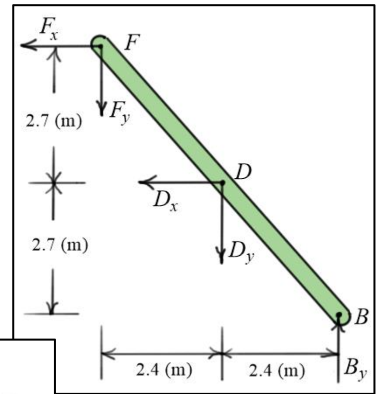


$$\circlearrowleft\!+\ \sum M_C = 2.4D_y - 3.6(2400) = 0 \quad \Rightarrow \boxed{D_y = 3600\ (\text{N})}$$

$$\circlearrowleft\!+\ \sum M_D = -2.4C_y - 1.2(2400) = 0 \quad \Rightarrow \boxed{C_y = -1200\ (\text{N})}$$

$$\boxed{\sum F_x = C_x + D_x = 0}$$

Member *BDF*:



$$\sum F_y = B_y - D_y - F_y = 0 \quad \Rightarrow F_y = B_y - D_y = 1800 - 3600$$

$$\Rightarrow \boxed{F_y = -1800 \text{ (N)}}$$

$$\circlearrowleft\!\!+ \sum M_D = 2.4B_y + 2.4F_y + 2.7F_x = 0$$

$$\Rightarrow F_x = -\tfrac{2.4}{2.7}\left(B_y + F_y\right) = -\tfrac{2.4}{2.7}\left(1800 - 1800\right)$$

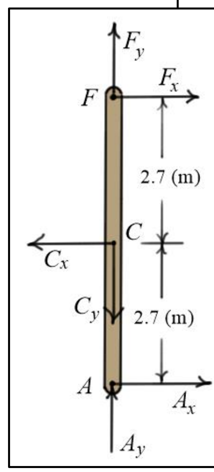
$$\Rightarrow \boxed{F_x = 0}$$

$$\sum F_x = -F_x - D_x = 0 \quad \Rightarrow \boxed{D_x = 0}$$

Member *CDE*:

$$C_x + D_x = 0 \quad \Rightarrow \boxed{C_x = 0}$$

Member *ACF*: (check)

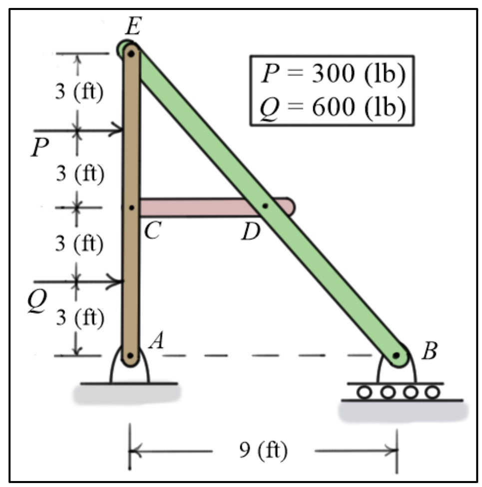


$$\boxed{\sum F_y = A_y + F_y - C_y = 600 - 1800 - \left(-1200\right) = 0} \quad ✓$$

Example #2:

Given: Frame loaded as shown

Find: All external support and member forces



Solution: (neglecting member weight forces)

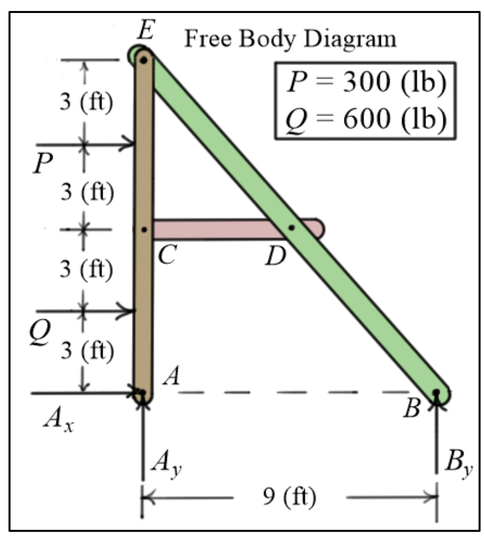
External Support Forces:

$$\circlearrowleft\!\!+ \sum M_A = 9B_y - 3Q - 9P = 0 \quad \Rightarrow B_y = \frac{3Q + 9P}{9}$$

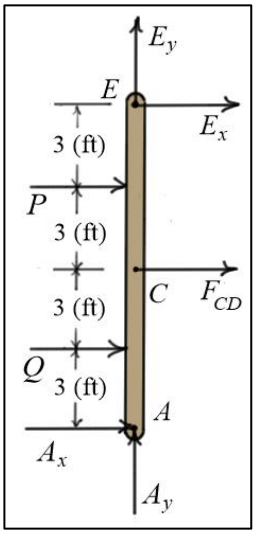
$$\Rightarrow \boxed{B_y = 500 \text{ (lb)}}$$

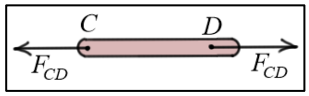
$$\sum F_y = A_y + B_y = 0 \quad \boxed{A_y = -B_y = -500 \text{ (lb)}}$$

$$\sum F_x = A_x + Q + P = 0 \quad \Rightarrow \boxed{A_x = -\left(Q + P\right) = -900 \text{ (lb)}}$$

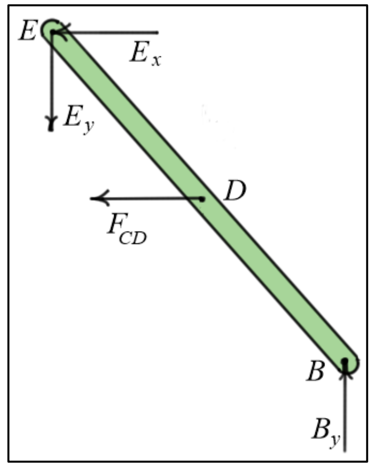


Member Free Body Diagrams:





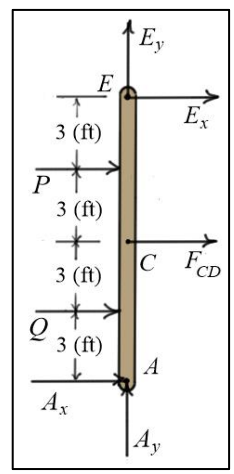
two-force member



Notes:

- Member CD is a two-force member. It must either be in tension or compression.
- Consistent with Newton's third law, the pin forces on any two adjoining members are equal and opposite on the free body diagrams.

Member ACE:



$$\circlearrowleft\!+\ \sum M_E = 12A_x + 6F_{CD} + 3P + 9Q = 0 \quad F_{CD} = \frac{-(12A_x + 3P + 9Q)}{6}$$

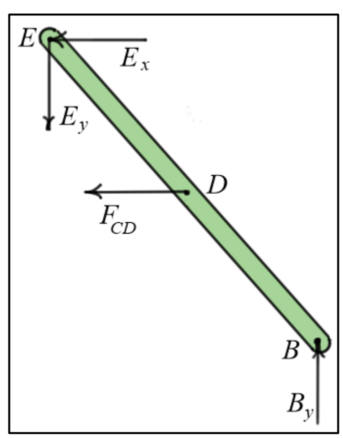
$$\Rightarrow \boxed{F_{CD} = 750 \text{ (lb)}}$$

$$\sum F_x = E_x + F_{CD} + P + Q + A_x = 0 \quad \Rightarrow E_x = -(F_{CD} + P + Q + A_x)$$

$$\Rightarrow \boxed{E_x = -750 \text{ (lb)}}$$

$$\sum F_y = A_y + E_y = 0 \quad \Rightarrow \boxed{E_y = -A_y = 500 \text{ (lb)}}$$

Member BDE: (check)



$$\boxed{E_x = -750 \text{ (lb)}}$$

$$\boxed{E_y = 500 \text{ (lb)}}$$

$$\boxed{F_{CD} = 750 \text{ (lb)}}$$

$$\boxed{B_y = 500 \text{ (lb)}}$$

$$\boxed{\sum F_x = -E_x - F_{CD} = 750 - 750 = 0} \checkmark$$

$$\boxed{\sum F_y = B_y - E_y = 500 - 500 = 0} \checkmark$$

$$\circlearrowleft\!+\ \sum M_B = 12E_x + 9E_y + 6F_{CD}$$

$$= 12(-750) + 9(500) + 6(750)$$

$$= -9000 + 4500 + 4500$$

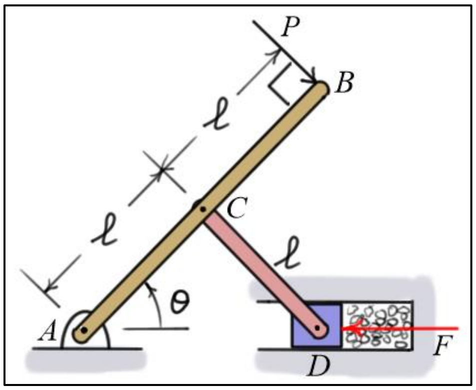
$$\Rightarrow \boxed{\sum M_B = 0} \checkmark$$

Example #3:

Given: The rock crushing machine loaded as shown.

P is a known applied load, and θ is a known angle.

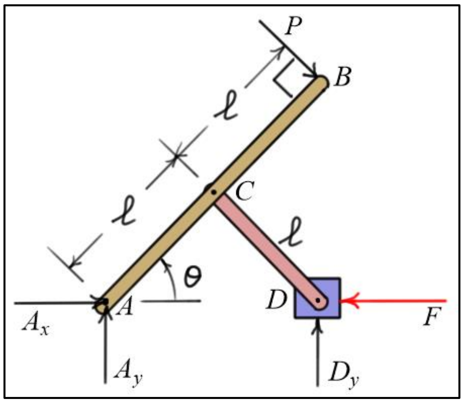
Find: Forces acting on member CD.



Solution:

External Support Forces:

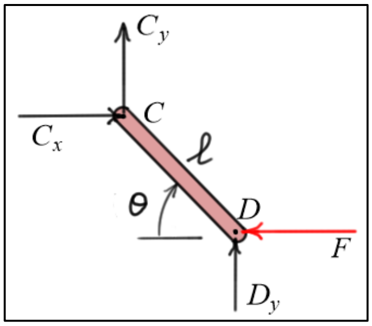
Note from the free body diagram there are four unknown external forces, but only three independent equations of equilibrium for the machine. So, to find all the external forces, the members of the machine must be separated.



$$\circlearrowleft\!+\;\sum M_A = \big(2\ell\cos(\theta)\big)D_y - \big(2\ell\big)P = 0 \quad \Rightarrow D_y = \frac{2\ell P}{2\ell\cos(\theta)}$$

$$\Rightarrow \boxed{D_y = \frac{P}{\cos(\theta)}}$$

Member CD:



$$\circlearrowleft\!+\;\sum M_C = \big(\ell\cos(\theta)\big)D_y - \big(\ell\sin(\theta)\big)F = 0 \quad \Rightarrow F = \left(\frac{\cos(\theta)}{\sin(\theta)}\right)D_y$$

$$\Rightarrow \boxed{F = \left(\frac{\cancel{\cos(\theta)}}{\sin(\theta)}\right)\left(\frac{P}{\cancel{\cos(\theta)}}\right) = \frac{P}{\sin(\theta)}}$$

$$\sum F_x = C_x - F = 0 \quad \Rightarrow \boxed{C_x = F = \frac{P}{\sin(\theta)}}$$

$$\sum F_y = C_y + D_y = 0 \quad \boxed{C_y = -D_y = \frac{-P}{\cos(\theta)}}$$

Note: For angles $0 < \theta < 90$ (deg), $0 < \sin(\theta) < 1$ and the working force $F > P$.

Example #4:

Given: Pliers loaded as shown.

Find: Gripping force F

Solution:

Member BC:

$$\circlearrowleft\!+\ \sum M_C = 160(250) - 35F = 0$$

$$\Rightarrow F = \frac{160(250)}{35}$$

$$\Rightarrow \boxed{F \approx 1143 \text{ (N)}}$$

$$\sum F_x = C_x - F\sin(30) = 0$$

$$\Rightarrow \boxed{C_x \approx 571 \text{ (N)}}$$

$$\sum F_y = C_y - F\cos(30) - 250 = 0$$

$$\Rightarrow \boxed{C_y \approx 1240 \text{ (N)}}$$

Similar equations apply for member AC.