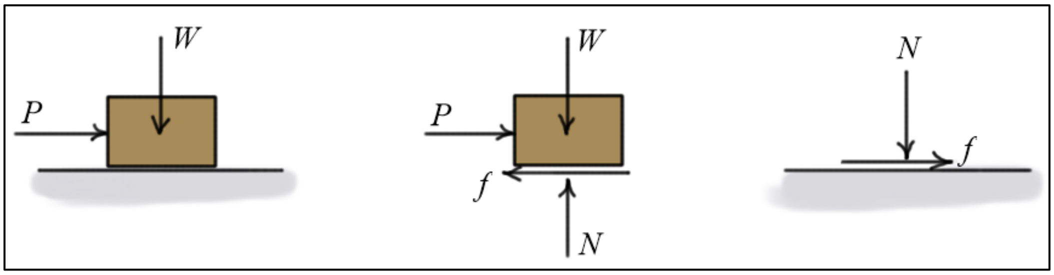
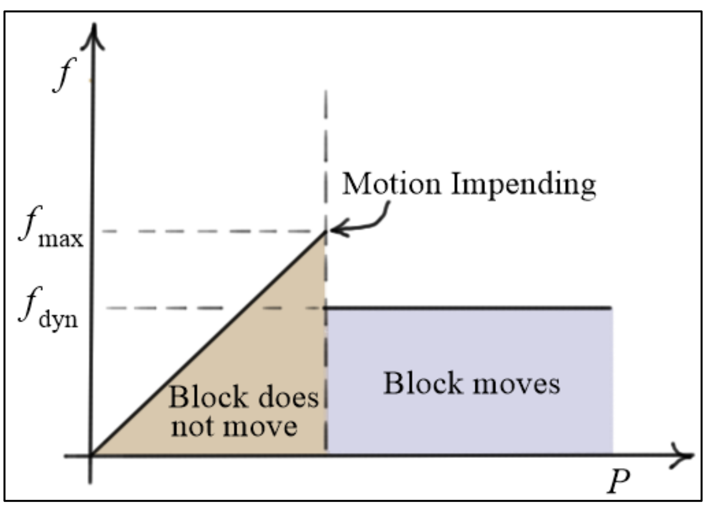
Elementary Statics
Dry Friction

- When two bodies are in ***contact***, the ***reactive contact force*** can be broken into ***two components***, one ***normal*** and one ***tangent*** to the plane of contact.
- The component of the contact force ***tangent*** to the plane of contact is called the ***friction force*** (f), and it is the result of the ***relative roughness*** of the contacting surfaces.
- Consider a ***block*** resting on a ***horizontal plane*** with an ***applied force*** P.



- Depending on the ***magnitude*** of P and the ***roughness*** of the contacting surfaces, the block ***may*** or ***may not*** move.
- There are ***three*** distinct ***possibilities*** in this case.
  - the block ***slides***
  - the block ***does not slide***, even when P is increased slightly
  - the block ***does not slide***, but it does slide when P is increased slightly
- In the last case, ***motion*** is ***impending***, because if P is increased slightly, the block will move.
- One ***model*** of ***friction*** is described by the plot below. As P is ***increased*** from zero, the friction force ***matches it*** up to a limiting value of $f_{\max}$. As P is ***increased beyond this value***, the ***friction force drops*** to its dynamic value f_{dyn} which is ***approximately constant***.



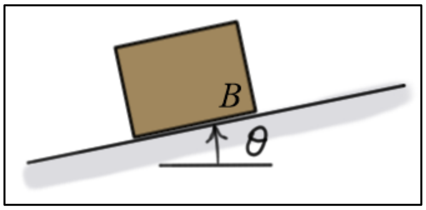
- The values of f_{max} and f_{dyn} are given in terms of the ***normal force*** N and the ***coefficients*** of ***static*** and ***kinetic friction***.

$$\boxed{f_{\text{max}} = \mu_s N} \qquad \mu_s \text{ is the } \textbf{\textit{coefficient}} \text{ of } \textbf{\textit{static friction}}$$

$$\boxed{f_{\text{dyn}} = \mu_k N} \qquad \mu_k \text{ is the } \textbf{\textit{coefficient}} \text{ of } \textbf{\textit{kinetic friction}}$$

- The ***coefficients*** of ***friction*** (μ_s and μ_k) depend on the ***relative roughness*** of the two surfaces. See your textbook or other reference sources for some ***typical values***.

<u>Example #1</u>:

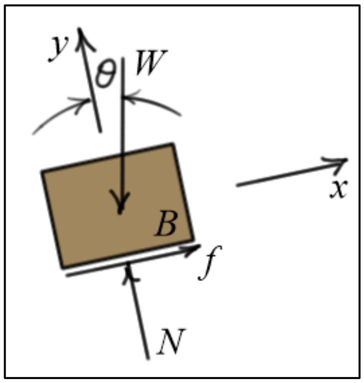


Given: Weight, W and coefficient of static friction μ_s.

Find: Maximum angle θ for which block B will remain in equilibrium.

Solution:

For this limiting case, we assume that motion is impending down the plane $\left(f = \mu_s N \right)$ and write the equilibrium equations.



$$+\nwarrow \sum F_y = N - W\cos(\theta) = 0 \quad \Rightarrow \boxed{N = W\cos(\theta)}$$

$$+\nearrow \sum F_x = f - W\sin(\theta) = 0 \quad \Rightarrow \boxed{\mu_s N - W\sin(\theta) = 0}$$

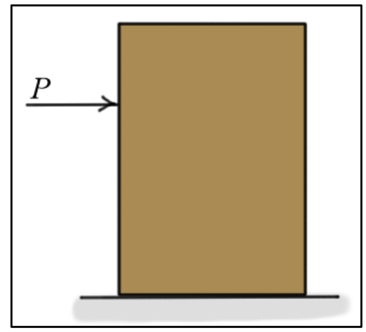
Combining the two boxed equations gives

$$\mu_s W\cos(\theta) - W\sin(\theta) = 0 \quad \Rightarrow \boxed{\theta = \tan^{-1}(\mu_s)}$$

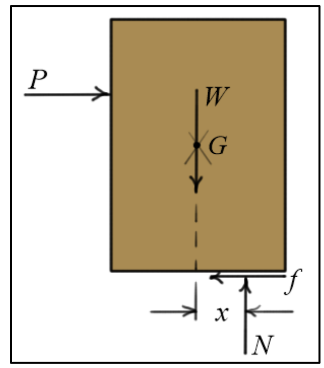
<u>Specific case</u>: If $\boxed{\mu_s = 0.4}$, then $\boxed{\theta = \tan^{-1}(0.4) \approx 21.8 \text{ (deg)}}$.

- If $\boxed{\theta < 21.8 \text{ (deg)}}$, the block will not move and $\boxed{f < f_{\text{max}} = \mu_s N}$
- If $\boxed{\theta = 21.8 \text{ (deg)}}$, the block will not move and $\boxed{f = f_{\text{max}} = \mu_s N}$ (motion impending)
- If $\boxed{\theta > 21.8 \text{ (deg)}}$, the block will slide down the plane and $\boxed{f = f_{\text{dyn}} = \mu_k N}$

Tipping Problems



- Consider the box resting on a plane with ***horizontal force*** $P = 0$. If the ***weight*** within the box is ***evenly distributed***, the ***normal force*** exerted by the ground on the box is ***uniformly distributed along the bottom*** of the box.
- As the value of P is increased $(P > 0)$, the ***normal distributed force*** exerted by the ground on the box is ***skewed*** either ***to the right*** or ***to the left*** of center.



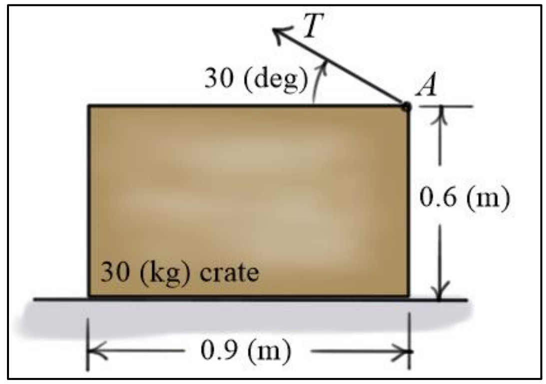
- If the box is in ***static equilibrium*** ($P = f$), the ***normal force*** will always be ***skewed*** to the ***right of center***.
  - If P is ***above*** the ***mass center*** G, both P and f will have ***clockwise moments*** about G which will be ***balanced*** by a ***counterclockwise moment*** from N.
  - If P is ***below*** the ***mass center*** and above the bottom of the box, P will have a ***counterclockwise moment*** about G, and f will have a ***larger clockwise moment*** about G. This ***net clockwise moment*** will be balanced by a ***counterclockwise moment*** of N. (Note: The friction force f has a ***larger moment arm*** about G than does P.)
- If the box is ***not*** in ***static equilibrium*** ($P > f$), the ***normal force*** will be ***skewed to the right of center*** if P is ***above*** G, and it ***can*** be ***skewed left*** or ***right of center*** if P is ***below*** G, depending on the relative magnitudes of P and f.
- Consider the case of static equilibrium where the ***applied force*** P is ***above*** the mass center, and the ***resultant normal force*** (N) is to the ***right*** of ***center*** as shown in the free-body diagram.
- In this case, the moments of the ***applied force*** P and the ***friction force*** f will both have a ***tendency*** to ***tip*** the box to the right, and the moment of ***normal force*** N will ***counter*** that.
- If N acts at the ***edge of the box*** to counter the effects of P and f, the box will be on the ***verge*** of ***tipping***.
- If N needs to act beyond the ***edge of the box*** to counter P and f, the box will ***tip***.

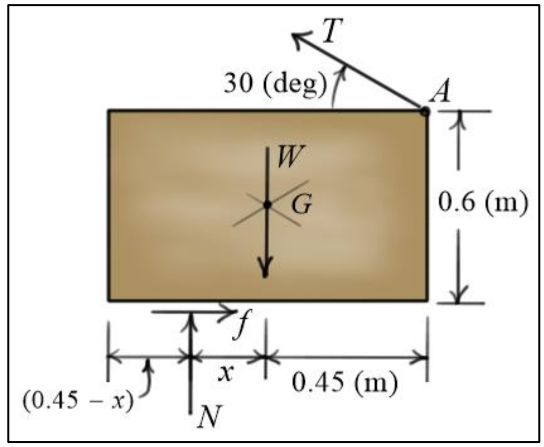
Example #2:

Given: 30 (kg) crate loaded as shown with $\mu_s = 0.35$

Find: a) tension T required to move the crate

b) whether the crate slides or tips





Solution:

a) To find the minimum tension required to move the crate, assume motion is impending and $\boxed{f = \mu_s N = 0.35N}$.

Force equations:

$$\overset{+}{\rightarrow}\sum F_x = f - T\cos(30) = 0$$

$$+\uparrow\sum F_y = N + T\sin(30) - mg = 0$$

Simultaneous equations:

$$\boxed{\begin{aligned} 0.35N - (\cos(30))T &= 0 \\ N + (\sin(30))T &= 30(9.81) \end{aligned}}$$

Solving gives:

$$\boxed{\begin{aligned} N &\approx 244.827 \approx 245 \text{ (N)} \\ T &\approx 98.9457 \approx 98.9 \text{ (N)} \end{aligned}}$$

Moment equation:

$$\circlearrowleft\!+\ \sum M_A = (0.45)mg + 0.6f - (0.45 + x)N = 0$$

$$\Rightarrow (0.45 + x)N = (0.45)mg + 0.6(0.35)N$$

$$\Rightarrow x = \frac{(0.45)mg}{N} + 0.6(0.35) - 0.45$$

$$\Rightarrow \boxed{x \approx 0.3009 \approx 0.301 \text{ (m)}}$$

b) The crate will ***slide*** (and not tip) because when motion is impending, N is found to be located between the centerline and edge of the crate, that is, $\boxed{x \approx 0.301 \text{ (m)} \le 0.45 \text{ (m)}}$. ***Note***: If x was found to be greater than 0.45 when motion is impending, the crate would tip and not slip.