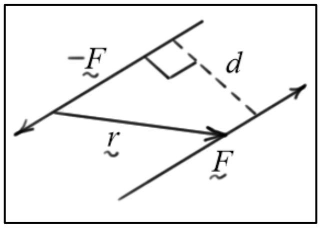
Elementary Statics
Equation Sheet #4: Moments of Couples/Equivalent Force Systems/Rigid Body Equilibrium



Moment of a Couple: $M_C = F \cdot d$ $\underset{\sim}{M}_C = \underset{\sim}{r} \times \underset{\sim}{F}$

Equivalent Force Systems

$$\underset{\sim}{F} = \underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i \quad \text{(acting at O)}$$

$$\underset{\sim}{M}_O = \sum_{\text{forces } (i)} \left(\underset{\sim}{r}_i \times \underset{\sim}{F}_i\right) + \sum_{\text{couples } (i)} \left(\underset{\sim}{M}_C\right)_i$$

Centroids of Areas

- Area, A: $\bar{x} = \frac{1}{A}\int_A x\, dA$ $\bar{y} = \frac{1}{A}\int_A y\, dA$
- Composite Area: $\bar{x} = \frac{\sum A_i\, \bar{x}_i}{\sum A_i}$ $\bar{y} = \frac{\sum A_i\, \bar{y}_i}{\sum A_i}$

Equivalent Force Systems for Distributed Loads

$$F_R = \sum F = \int_0^L w(x)\, dx \qquad \bar{x} = \frac{1}{F_R}\int_0^L x\, w(x)\, dx$$

Rigid Body Equilibrium

Vector Equations:

$$\underset{\sim}{F}_R = \sum_i \underset{\sim}{F}_i = \underset{\sim}{0}$$

$$\underset{\sim}{M}_P = \sum_i \left(\underset{\sim}{p}_i \times \underset{\sim}{F}_i\right) = \underset{\sim}{0}$$

Scalar Equations in 2D:

$$\begin{array}{l}\sum F_x = 0\\ \sum F_y = 0\\ \sum M_P = 0\end{array} \quad \text{or} \quad \begin{array}{l}\sum F_x = 0 \;\; \text{-or-} \;\; \sum F_y = 0\\ \sum M_P = 0\\ \sum M_Q = 0\end{array} \quad \text{or} \quad \begin{array}{l}\sum M_P = 0\\ \sum M_Q = 0\\ \sum M_R = 0\end{array}$$

(P, Q, R not colinear)

Scalar Equations in 3D:

$$\begin{array}{l}\sum F_x = 0\\ \sum F_y = 0\\ \sum F_z = 0\end{array} \quad \text{and} \quad \begin{array}{l}\sum (M_P)_x = 0\\ \sum (M_P)_y = 0\\ \sum (M_P)_z = 0\end{array}$$