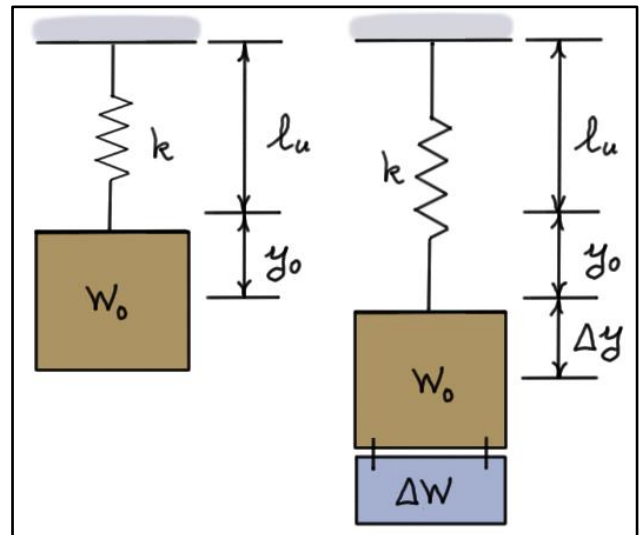


## Elementary Engineering Mathematics

### Application of Lines – Elementary Statics, Mechanics of Materials, and Dynamics

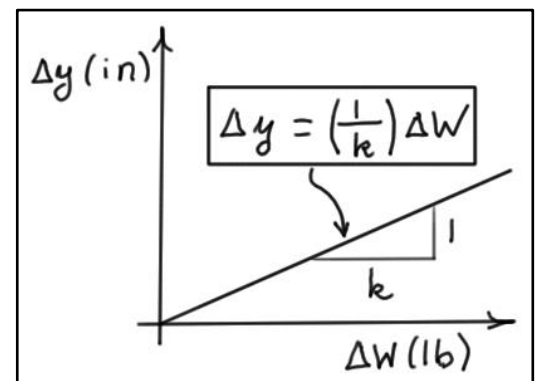
#### Example #1

Given: Consider a weight  $W_0 = 17.3 \text{ (lb)}$  which is supported by a **linear spring** of stiffness  $k$ . The length  $l_u$  is the **unstretched length** of the spring, and  $y_0$  is the required elongation of the spring to hold  $W_0$ . As additional weights ( $\Delta W$ ) are added, the spring stretches more ( $\Delta y$ ) to hold the additional weight. During this experiment, the additional displacement  $\Delta y$  can be related to the added weight  $\Delta W$  using the equation of a line.



By adding known weights to the system and measuring the subsequent displacement changes, the following data were collected:

| Weight, $\Delta W$ (lb) | Displacement, $\Delta y$ (in) |
|-------------------------|-------------------------------|
| 10                      | 1.21                          |
| 20                      | 2.45                          |



#### Find:

- estimate of the spring stiffness  $k$  (lb / ft),
- estimate of the initial displacement  $y_0$  (in), and
- find an equation for the total displacement  $y$  as a function of  $\Delta W$ .

#### Solution:

- a) The slope of the line is  $m = \frac{\Delta y}{\Delta W} = 1/k$ , so  $k = \frac{\Delta W}{\Delta y}$

| Weight, $\Delta W$ (lb) | Displacement, $\Delta y$ (in) | Stiffness, $k$ (lb / in) |
|-------------------------|-------------------------------|--------------------------|
| 10                      | 1.21                          | 8.2645                   |
| 20                      | 2.45                          | 8.1633                   |
| Average                 |                               | 8.214                    |

An estimate of the spring stiffness is the average derived from the two measurements.

$$\text{Units change: } k = \left[ \frac{8.214 \text{ (lb)}}{\text{(in)}} \right] \times \left[ \frac{12 \text{ (in)}}{\text{(ft)}} \right] = 98.57 \text{ (lb / ft)}$$

- b) The initial displacement  $y_0$  can be found by noting that the initial displacement and weight are related by the same stiffness.

$$y_0 = W_0 / k = 17.3 \text{ (lb)} / 8.214 \text{ (lb / in)} = 2.106 \text{ (in)}$$

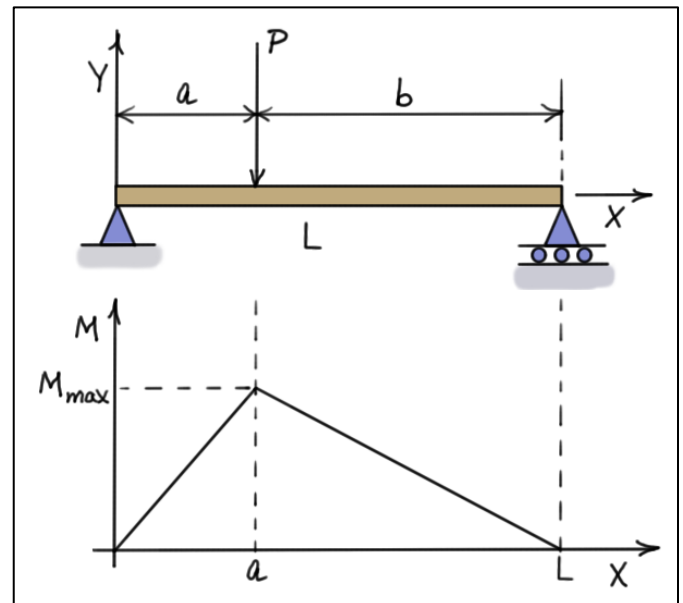
- c) The equation for the total displacement can be found using the *slope-intercept* form.

$$y = 2.106 + (1/8.214)\Delta W = 2.106 + (0.1217)\Delta W$$

### Example #2

Given: Consider a *long slender beam* of length  $L$  with a *concentrated load*  $P$  acting at distance  $a$  from the left end. Due to this load, the beam experiences an *internal bending moment*  $M$  that varies linearly across the length of the beam as shown. The maximum bending moment  $M_{\max}$  occurs at the load.

In an experiment, a load  $P = 100 \text{ (lbs)}$  is applied to a beam of length  $L = 5 \text{ (ft)}$ . The bending moments measured at two points on either side of  $P$  are



| Location, $x \text{ (ft)}$ | Moment, $M \text{ (ft - lb)}$ | Location Relative to Load |
|----------------------------|-------------------------------|---------------------------|
| 2.067                      | 64.3                          | left of load              |
| 4.378                      | 42.87                         | right of load             |

Find:

- a) the *moment equations* for  $0 \leq x \leq a$  and  $a \leq x \leq L$ ; b) the *location* of the load  $P$ ; and c) the *maximum moment* experienced by the beam.

Solution:

a) For  $0 \leq x \leq a$ , the slope of the line is  $m = (64.3 - 0)/(2.067 - 0) = 31.11$ , so

$$M(x) = mx = 31.11x \quad (1)$$

For  $a \leq x \leq L$ , the slope of the line is  $m = (0 - 42.87)/(5 - 4.378) = -68.92$ . Using the point-slope form of a line, we can write

$$(M - 42.87) = -68.92(x - 4.378) \Rightarrow M = (42.87 + (68.92 \times 4.378)) - 68.92x$$

or

$$M(x) = 344.6 - 68.92x \quad (2)$$

b) The load  $P$  is located at the point  $x = a$  where the moment equations (1) and (2) are equal, that is, at the *intersection* of the two lines. To find  $a$ , set

$$31.11x = 344.6 - 68.92x \Rightarrow (31.11 + 68.92)x = 344.6$$

or

$$a = 344.6 / (31.11 + 68.92) = 3.445 \text{ (ft)}$$

c) The *maximum moment* experienced by the beam can be calculated by substituting the value of  $a$  into equations (1) or (2).

$$M_{\max} = M(a) = 31.11 \times 3.445 = 107.2 \text{ (ft-lb)}$$

or

$$M_{\max} = M(a) = 344.6 - (68.92 \times 3.445) = 107.2 \text{ (ft-lb)}$$

