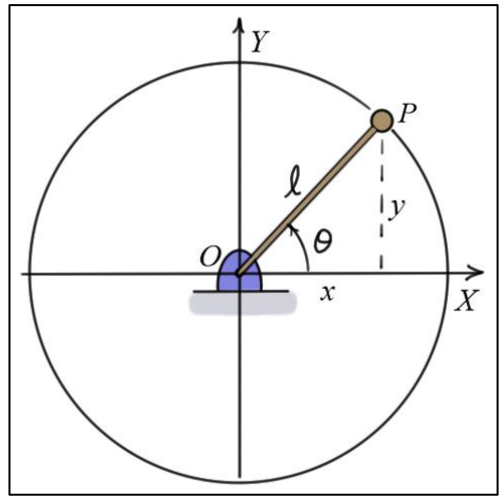
Elementary Engineering Mathematics
Sine and Cosine Functions of Time

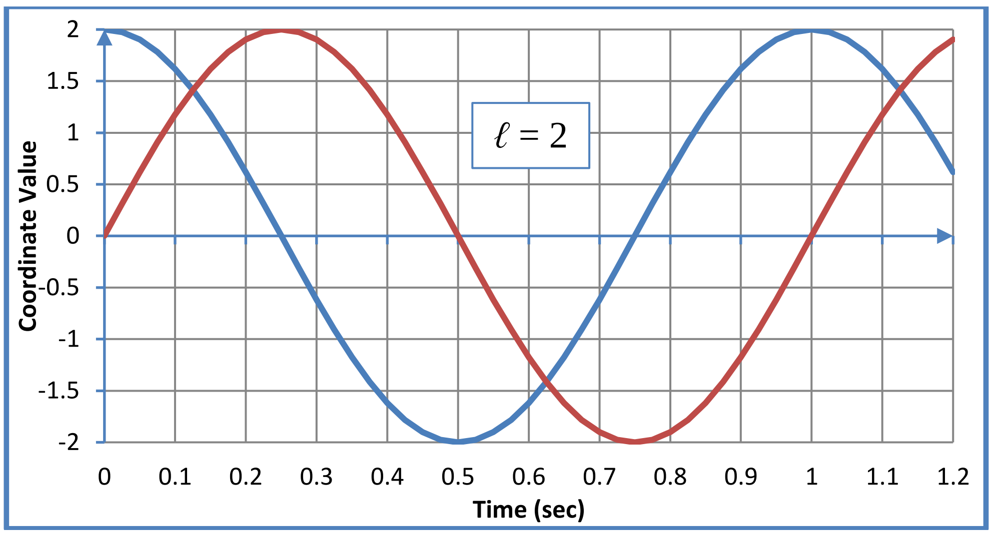


Arm OP rotates so P moves in a circular path. From trigonometry, the coordinates of P are

$$x = \ell\cos(\theta) \qquad y = \ell\sin(\theta)$$

If the bar completes one revolution (2π radians) in one second, then $\theta = 2\pi t$. So, we can also express the coordinates as functions of time.

$$x = \ell\cos(2\pi t) \qquad y = \ell\sin(2\pi t)$$



Characteristic	Symbol
Amplitude	$A = \ell$
Frequency	$\omega = 2\pi$ (radians/sec)
	$f = \omega/2\pi$ (cycles/sec) -or- (Hertz (Hz))
Period	$T = 1/f$ (seconds/cycle)

Note that the sine and cosine functions (and hence the X and Y coordinates) can be related by a ***phase shift***. The phase shift corresponds to a ***shift*** in ***time***.

$$\sin(2\pi t) = \cos\left(2\pi t - \pi/2\right) \qquad \cos(2\pi t) = \sin\left(2\pi t + \pi/2\right)$$

In this case, the phase shift of $\pi/2$ radians equates to a shift in time of $\frac{1}{4}$ second. For any given phase shift, the corresponding time shift depends on the frequency ω.