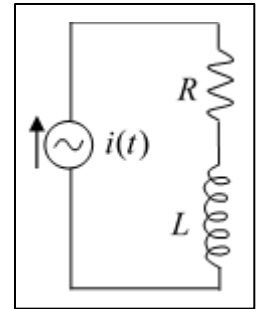


## Elementary Engineering Mathematics

### Application of Sine and Cosine Functions in Electrical Engineering

Given: A current source  $i(t) = 2 \sin(120\pi t)$  is applied to an  $RL$  series circuit with  $R = 100$  (ohms) and  $L = 100$  (mh). The voltage drop across the resistor is  $V_R = Z_R I$ , the voltage drop across the inductor is  $V_L = Z_L I$ , and the total voltage drop across both elements is  $V = V_R + V_L$ .



Find: (a)  $v(t)$  the voltage as a function of time. (b)  $v(t)$  as a single, phase-shifted *sine wave*.  
(c)  $v(t)$  as a single, phase-shifted *cosine wave*.

Solution:

$$\text{a) } V_R = Z_R I = (100 \angle 0^\circ)(2 \angle -90^\circ) = 200 \angle -90^\circ$$

$$\text{or } v_R(t) = 200 \cos(120\pi t - 90) = 200 \sin(120\pi t)$$

$$V_L = Z_L I = ((120\pi(0.1)) \angle 90^\circ)(2 \angle -90^\circ) \approx 75.4 \angle 0^\circ \quad \text{or } v_L(t) \approx 75.4 \cos(120\pi t)$$

So,

$$v(t) \approx 200 \sin(120\pi t) + 75.4 \cos(120\pi t)$$

$$\text{b) } v(t) = M \sin(120\pi t + \phi)$$

$$M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)} \quad \phi \approx \tan^{-1}(75.4/200) \approx 20.66^\circ \approx 0.3605 \text{ (rad)}$$

$$\text{c) } v(t) = M \cos(120\pi t + \phi)$$

$$M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)} \quad \phi \approx \tan^{-1}(-200/75.4) \approx -69.34^\circ \approx -1.21 \text{ (rad)}$$

Alternate Solution:

$$V_R = Z_R I = 200 \angle -90^\circ = -j200 \quad \text{and} \quad V_L = Z_L I \approx 75.4 \angle 0^\circ \approx 75.4$$

So,

$$V = 75.4 - j200 = \left(\sqrt{75.4^2 + 200^2}\right) \angle \tan^{-1}(-200/75.4) \approx 214 \angle (-1.21) \text{ (volts)}$$

Or,

$$\begin{aligned} v(t) &= 214 \cos(120\pi t - 1.21) = 214 \sin(120\pi t - 1.21 + \pi/2) \\ &= 214 \sin(120\pi t + 0.3605) \text{ (volts)} \end{aligned}$$