

Elementary Engineering Mathematics

Equation Sheet #3 – Systems of Equations & Exponential, Sine, Cosine Functions

Systems of Equations and Matrices

1. Linear Algebraic Equations: $[A]\{x\} = \{b\}$

$$[A]_{n \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \{x\} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \{b\} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

2. Cramer's Rule for $n = 2$

$$x_1 = \frac{\det \begin{bmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{bmatrix}}{\det [A]} \quad x_2 = \frac{\det \begin{bmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{bmatrix}}{\det [A]} \quad \det [A] = a_{11}a_{22} - a_{12}a_{21}$$

3. Matrix Inversion

$$[A]\{x\} = \{b\} \Rightarrow \{x\} = [A]^{-1}\{b\} \quad [A]^{-1} \text{ exists only if } \det [A] \neq 0$$

For 2×2 matrices:

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \Rightarrow [A]^{-1} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} / \det [A] = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} / (a_{11}a_{22} - a_{12}a_{21})$$

Exponential, Natural Logarithms, Sine, and Cosine Functions

1. Exponential and Natural Log

$$f(x) = e^x \quad (e = 2.71828\dots)$$

$$\text{Natural Log is the Inverse Function: } \log_e(e^x) = \ln(e^x) = x \quad e^{\log_e(x)} = e^{\ln(x)} = x$$

2. Useful Sine and Cosine Identities

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

3. Sine and Cosine Combinations

$$x(t) = A \sin(\omega t) + B \cos(\omega t) = M \sin(\omega t + \phi) \quad M = \sqrt{A^2 + B^2} \quad \phi = \tan^{-1}(B/A)$$

$$x(t) = A \sin(\omega t) + B \cos(\omega t) = M \cos(\omega t + \phi) \quad M = \sqrt{A^2 + B^2} \quad \phi = \tan^{-1}(-A/B)$$

$$\text{Frequency: } \omega \text{ (rad/s)} \quad f = \frac{\omega}{2\pi} \text{ (Hz)} \quad \text{Period: } T = \frac{1}{f} \text{ (s/cycle)}$$