Elementary Engineering Mathematics
Exercises #8 – Derivatives

1. The stiffness (k) of a spring at a given displacement (x_0) is the slope (derivative) of the force-displacement curve measured at that displacement. Given a hardening spring having the force-displacement curve $F = f(x) = 100 + 10x + x^2$ (lb), (a) use the limiting process discussed in the notes to find the stiffness of the spring at $x_0 = 3$ (in), (b) find a linear approximation of $f(x)$ about $x_0 = 3$ (in), and (c) calculate the percent error of the approximation over the range of x shown in the table.

x (in)	F_{approx}	F_{actual}	% error
1			
2			
3			
4			
5			

$$f'(x_0) = \lim_{h\to 0}\left(\frac{f(x_0+h) - f(x_0)}{h}\right)$$

$$\% \text{ error} = 100\left(F_{approx} - F_{actual}\right)/F_{actual}$$

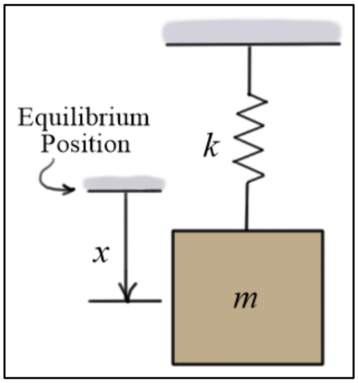
2. The acceleration of an object at a given time (t_0) is the derivative of the velocity at that time. Given a car with velocity defined by the function $v(t) = f(t) = 36/t$ (m/sec), (a) use the limiting process discussed in the notes to find the acceleration of the car (derivative of $f(t)$) at $t_0 = 2$ (sec), (b) find a linear approximation of $f(t)$ about $t_0 = 2$ (sec), and (c) calculate the percent error of the approximation over the range of t shown in the table.

t (sec)	v_{approx}	v_{actual}	% error
1.5			
1.75			
2			
2.25			
2.5			

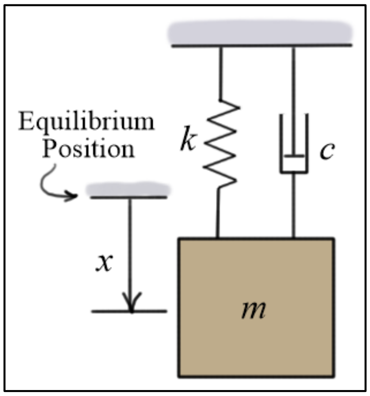
$$\% \text{ error} = 100\left(v_{approx} - v_{actual}\right)/v_{actual}$$

3. The trajectory of a ball that was thrown into the air is defined by the two functions, $x(t) = 15t$ and $y(t) = 5 + 26t - 4.905t^2$. The variables $x(t)$ and $y(t)$ measure the horizontal and vertical positions of the ball in meters as a function of time (measured in seconds). Using the table of derivatives and the rules for differentiation, find (a) $\dot{x}(t)$ and $\ddot{x}(t)$ the first and second time derivatives of $x(t)$, (b) $\dot{y}(t)$ and $\ddot{y}(t)$ the first and second time derivatives of $y(t)$, (c) the slope of $y(t)$ when $t = 4$ (sec), (d) $\underset{\sim}{V}$ the velocity vector of the ball at $t = 4$ (sec), and (e) the X and Y coordinates of the ball when it reaches its maximum vertical position.

4. For the spring-mass system shown, the spring stiffness is $k = 25$ (lb/ft), the mass is $m = 0.25$ (slug), the initial position is $x_0 = 0.5$ (ft), and the initial velocity is $v_0 = 15$ (ft/s). Using the table of derivatives and the rules for differentiation, differentiate the displacement function $\boxed{x(t) = \frac{v_0}{\omega}\sin(\omega t) + x_0 \cos(\omega t)}$ with $\boxed{\omega = \sqrt{\frac{k}{m}}}$ to find, (a) $v(t) = \dot{x}(t)$ the velocity of the mass, and (b) $a(t) = \ddot{x}(t)$ the acceleration of the mass. Using these results, find (c) a_0 the initial acceleration of the mass, and (d) the times when the velocity is maximum or minimum.



5. For the over-damped spring-mass-damper system shown, the spring stiffness is $k = 25$ (lb/ft), the damping coefficient is $c = 7$ (lb-s/ft), the mass is $m = 0.25$ (slug), the initial position is $x_0 = 0.5$ (ft), and the initial velocity is $v_0 = 15$ (ft/s). Using the table of derivatives and the rules for differentiation, differentiate the displacement function to find (a) $v(t) = \dot{x}(t)$ the velocity, and (b) $a(t) = \ddot{x}(t)$ the acceleration of the mass. Using these results, find (c) a_0 the initial acceleration of the mass, and (d) the time(s) when the displacement is maximum.



6. In the simple capacitor circuit shown, the current is proportional to the derivative of the applied voltage, that is $i(t) = C\frac{dv}{dt}$. Given $C = 500\,(\mu\text{f})$ and $v(t) = 100\cos(240\pi t)$ (volts), find

   a) the current $i(t)$
   b) the power $p(t)$ and the maximum power p_{max}