

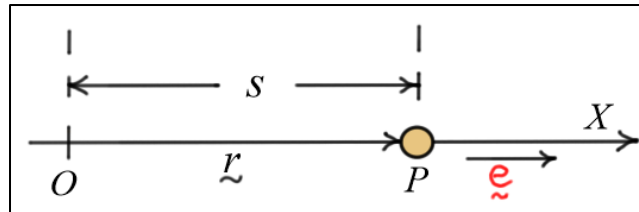
Elementary Dynamics

Rectilinear (Straight Line) Motion

General Concepts:

Position, Velocity, and Acceleration

A particle P has *rectilinear* motion when it moves in a straight line. As shown in the figure, define the direction of motion as the X -axis along which we define a *unit vector*, \underline{e} .



The *position vector* of P may then be written as $\underline{r} = s \underline{e}$, where s is the distance from some fixed point on the axis (in this case, O) to P . The *velocity* of P is defined as the derivative of the position vector. Using the *product rule*, we can write

$$\underline{v} = \frac{d\underline{r}}{dt} = \frac{d}{dt}(s\underline{e}) = \left(\frac{ds}{dt} \underline{e}\right) + \left(s \frac{d\underline{e}}{dt}\right) = \frac{ds}{dt} \underline{e} = \dot{s} \underline{e} = v \underline{e}$$

Similarly, the *acceleration* of P is defined as the derivative of the velocity. So, the acceleration may be written as

$$\underline{a} = \frac{d^2 s}{dt^2} \underline{e} = \ddot{s} \underline{e} = \dot{v} \underline{e} = a \underline{e}$$

Average Velocity, Average Speed, and Average Acceleration

If Δs and s_T represent the “displacement” and “total distance” traveled by P over the interval of time Δt , then the average velocity and average speed of P over the interval Δt are defined to be

$$\text{Average Velocity: } \underline{v}_{avg} = \frac{\Delta \underline{r}}{\Delta t}, \quad \text{Average Speed: } v_{avg} = \frac{s_T}{\Delta t}.$$

The average acceleration over the time interval Δt is defined to be: $\underline{a}_{avg} = \frac{\Delta \underline{v}}{\Delta t}$.

Problem Solving:

The following list provides a guide to solving rectilinear motion problems. Given a position, velocity, or acceleration function, the list shows what you can calculate.

1. Given: $s = s(t)$ Calculate: $v(t) = \frac{ds}{dt}$ and $a(t) = \frac{dv}{dt}$

2. Given: $v = v(t)$ Calculate: $\int_{s_0}^s ds = s(t) - s(0) = \int_{t_0}^t v(t)dt$ and $a(t) = \frac{dv}{dt}$

3. Given: $a = a(t)$ Calculate: $\frac{dv}{dt} = a(t) \Rightarrow \begin{cases} \int_{v_0}^v dv = v(t) - v(0) = \int_{t_0}^t a(t)dt \\ \int_{s_0}^s ds = s(t) - s(0) = \int_{t_0}^t v(t)dt \end{cases}$

4. Given: $a = a(s)$ Calculate: $v \frac{dv}{ds} = a(s) \Rightarrow \begin{cases} \int_{v_0}^v v dv = \frac{1}{2}(v^2 - v_0^2) = \int_{s_0}^s a(s) ds \\ \frac{ds}{dt} = v(s) \Rightarrow \int_{s_0}^s \frac{ds}{v(s)} = \int_{t_0}^t dt = t - t_0 \end{cases}$

5. Given: $a = a(v)$ Calculate: $v \frac{dv}{ds} = a(v) \Rightarrow \int_{v_0}^v \frac{v dv}{a(v)} = \int_{s_0}^s ds = s(v) - s(0)$

6. Given: $a = a(v)$ Calculate: $\frac{dv}{dt} = a(v) \Rightarrow \begin{cases} \int_{v_0}^v \frac{dv}{a(v)} = \int_{t_0}^t dt = t - t_0 \\ \int_{s_0}^s ds = s(t) - s(0) = \int_{t_0}^t v(t) dt \end{cases}$

7. Given: $a = a_0 = \text{constant}$

Calculate: $\frac{dv}{dt} = a_0 \Rightarrow \begin{cases} v(t) = v_0 + a_0(t - t_0) \\ s(t) = s_0 + v_0(t - t_0) + \frac{1}{2}a_0(t - t_0)^2 \end{cases}$

Calculate: $v \frac{dv}{ds} = a_0 \Rightarrow v^2(t) = v_0^2 + 2a_0(s - s_0)$