

Multibody Dynamics

Representation of Vector Operations as Matrix Operations

Dot (or Scalar) Product

Given two vectors \underline{a} and \underline{b} both expressed in terms of the unit vector set $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$

$$\underline{a} = a_1 \underline{e}_1 + a_2 \underline{e}_2 + a_3 \underline{e}_3$$

$$\underline{b} = b_1 \underline{e}_1 + b_2 \underline{e}_2 + b_3 \underline{e}_3$$

then the dot (or *scalar*) product of the two vectors is defined to be

$$\underline{a} \cdot \underline{b} = \sum_{i=1}^3 a_i b_i.$$

This product can be represented as the following *matrix product*

$$\underline{a} \cdot \underline{b} \rightarrow \{a\}^T \{b\}$$

Here, $\{a\}$ indicates a 3×1 *column vector* whose elements are the components of the vector \underline{a} ,

and $\{a\}^T$ indicates a 1×3 *row vector* that is the *transpose* of $\{a\}$. Recall for any two vectors \underline{a}

and \underline{b} , $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$. That is, the dot product is *commutative*. The matrix equivalent of this

statement is $\{a\}^T \{b\} = \{b\}^T \{a\} = (\{a\}^T \{b\})^T$.

Cross (or Vector) Product

Again, given two vectors \underline{a} and \underline{b} both expressed in terms of the same unit vector set

$$\underline{a} = a_1 \underline{e}_1 + a_2 \underline{e}_2 + a_3 \underline{e}_3$$

$$\underline{b} = b_1 \underline{e}_1 + b_2 \underline{e}_2 + b_3 \underline{e}_3$$

then the cross (or *vector*) product of the two vectors is defined to be

$$\underline{a} \times \underline{b} = (a_2 b_3 - a_3 b_2) \underline{e}_1 + (a_3 b_1 - a_1 b_3) \underline{e}_2 + (a_1 b_2 - a_2 b_1) \underline{e}_3.$$

This product can be represented as the following *matrix product*

$$\underline{a} \times \underline{b} \rightarrow [\tilde{a}] \{b\} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

Here, $[\tilde{a}]$ is a *skew-symmetric matrix* defined (as above) using the components of \underline{a} . Recall that for any two vectors \underline{a} and \underline{b} , $\underline{a} \times \underline{b} = -\underline{b} \times \underline{a}$. The matrix equivalent of this statement is $[\tilde{a}]\{b\} = -[\tilde{b}]\{a\}$. Recall also that a skew-symmetric matrix is *equal* to the *negative* of its *transpose*. So, for example, $[\tilde{a}]\{b\} = -[\tilde{a}]^T\{b\}$.