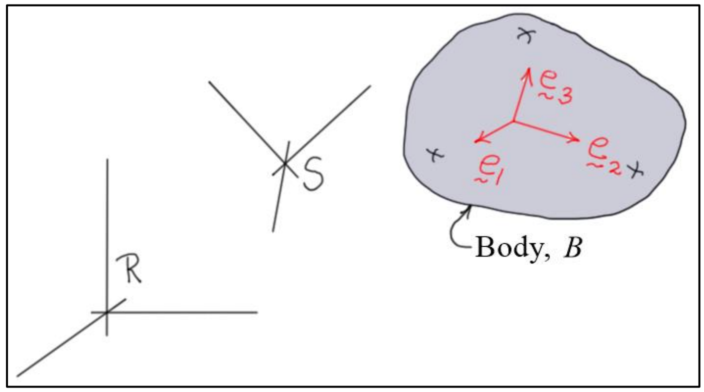
Intermediate Dynamics
Summation Rule for Angular Velocities

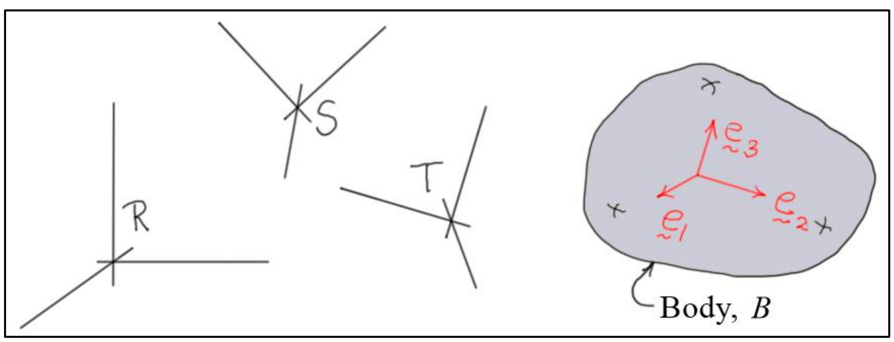
Consider a rigid body B undergoing ***three-dimensional motion*** as shown in the diagram below. R and S represent two ***reference frames*** that are ***rotating*** relative to each other. The ***angular velocity*** of the body B ***relative*** to the reference frame R (written as ${}^R\underset{\sim}{\omega}_B$) can be found by using the ***summation rule*** for angular velocities to work through the ***intermediate*** reference frame S as follows.



$$\boxed{{}^R\underset{\sim}{\omega}_B = {}^S\underset{\sim}{\omega}_B + {}^R\underset{\sim}{\omega}_S}$$

Here, ${}^S\underset{\sim}{\omega}_B$ represents the angular velocity of B relative to the reference frame S, and ${}^R\underset{\sim}{\omega}_S$ represents the angular velocity of frame S relative to R.

Consider next the body B in the the diagram below. Here, there are three reference frames, R, S, and T, all rotating relative to each other. In this case, ${}^R\underset{\sim}{\omega}_B$ the angular velocity of B relative to R can be found using the summation rule for angular velocities to work through the intermediate frames S and T as follows



$$\boxed{\begin{aligned} {}^R\underset{\sim}{\omega}_B &= {}^T\underset{\sim}{\omega}_B + {}^R\underset{\sim}{\omega}_T \\ &= {}^T\underset{\sim}{\omega}_B + {}^S\underset{\sim}{\omega}_T + {}^R\underset{\sim}{\omega}_S \end{aligned}}$$

In fact, this rule can be extended to as many frames as necessary.

The ***summation rule*** can be used to compute the angular velocity of a body (undergoing three-dimensional motion) by introducing a set of reference frames whose relative angular motions can be described using simple angular velocities. The angular velocity of the body is found by summing the simple angular velocities.

Note: There is ***no*** corresponding summation rule for ***angular accelerations***. The ***angular acceleration*** of a body is found by ***direct differentiation*** of the angular velocity vector.

That is, $\boxed{{}^R\underset{\sim}{\alpha}_B = \dfrac{{}^Rd}{dt}({}^R\underset{\sim}{\omega}_B)}$.