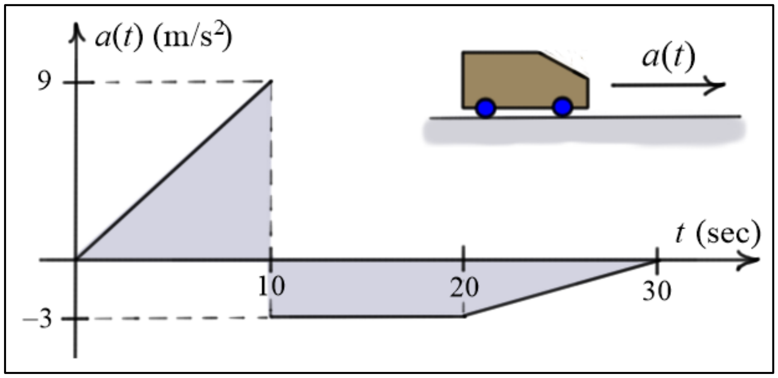
Elementary Dynamics
Acceleration Profiles – A Second Example

Given:

A car has the acceleration profile shown where $a(t)$ is in m/s^2.



Find:

$v(t)$ and $s(t)$ the speed and position of the car as functions of time for $0 \le t \le 30$ (sec). Also, find the speed and position of the car when $t = 30$ (sec). Assume $v(0) = s(0) = 0$.

Solution:

$\underline{0 \le t \le 10 \text{ (sec)}}$: $\quad a(t) = 0.9t = \dfrac{dv}{dt}, \quad v(t) = \dfrac{ds}{dt}$

$$\int_{v_0}^{v(t)} dv = \int_0^t 0.9t\,dt \quad \Rightarrow \quad \boxed{v(t) = 0.45t^2} \quad \Rightarrow \quad v(10) = 45 \text{ (m/sec)}$$

$$\int_{s_0}^{s(t)} ds = \int_0^t 0.45t^2\,dt \quad \Rightarrow \quad \boxed{s(t) = 0.15t^3} \quad \Rightarrow \quad s(10) = 150 \text{ (m)}$$

$\underline{10 \le t \le 20 \text{ (sec)}}$: $\quad a(t) = -3 = \dfrac{dv}{dt}, \quad v(t) = \dfrac{ds}{dt}$

$$\int_{v(10)}^{v(t)} dv = \int_{10}^t -3dt \quad \Rightarrow \quad \boxed{v(t) = 75 - 3t} \quad \Rightarrow \quad v(20) = 15 \text{ (m/sec)}$$

$$\int_{s(10)}^{s(t)} ds = \int_{10}^t (75 - 3t)dt \quad \Rightarrow \boxed{s(t) = -\tfrac{3}{2}t^2 + 75t - 450} \quad \Rightarrow s(20) = 450 \text{ (m)}$$

$\underline{20 \le t \le 30 \text{ (sec)}}$: $\quad a(t) = 0.3t - 9 = \dfrac{dv}{dt}, \quad v(t) = \dfrac{ds}{dt}$

$$\int_{v(20)}^{v(t)} dv = \int_{20}^t (0.3t - 9)dt \quad \Rightarrow \boxed{v(t) = 0.15t^2 - 9t + 135} \quad \Rightarrow \boxed{v(30) = 0 \text{ (m/sec)}}$$

$$\int_{s(20)}^{s(t)} ds = \int_{20}^t (0.15t^2 - 9t + 135)\,dt \quad \Rightarrow \boxed{s(t) = 0.05t^3 - 4.5t^2 + 135t - 850}$$

$$\Rightarrow \boxed{s(30) = 500 \text{ (m)}}$$

Plots of the position, velocity and acceleration functions for $0 \le t \le 30$ (sec) are shown below. Note the ***areas*** under the ***acceleration diagram*** give the ***changes*** in the ***velocity***, and the ***areas*** under the ***velocity diagram*** give the ***changes*** in the ***position***. The acceleration diagram is discontinuous, the velocity diagram is piecewise continuous, and the position diagram is continuous.