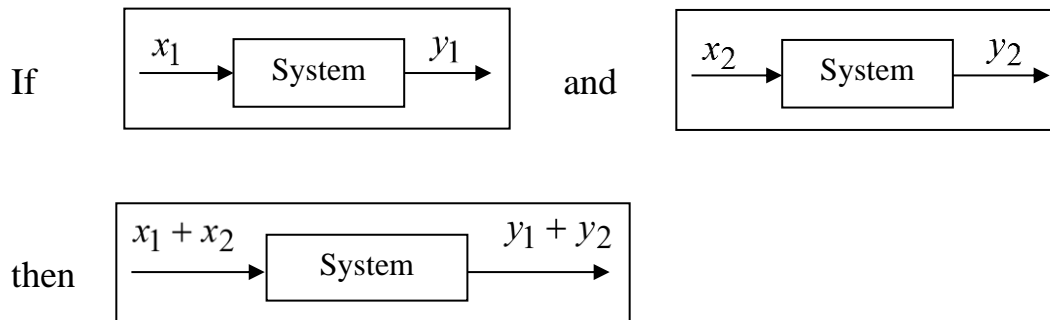


Introductory Control Systems

Linearity Principles

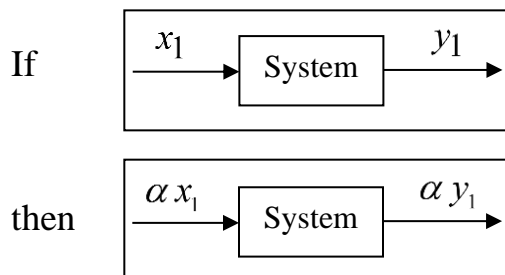
- The analysis of nonlinear systems is, in general, quite complicated – *solutions to nonlinear equations are not easily obtained*.
- However, if these systems remain *close* to some *nominal operating conditions*, then it may be appropriate to analyze their behavior using a model that has been *linearized* about the nominal conditions. This *simplifies* the analysis of the system behavior.
- *Two properties* must be satisfied by a system (or model) for it to be linear. These properties are defined in terms of the system input and output as follows.

1. Principle of Superposition



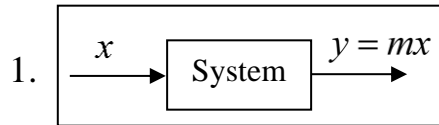
That is, if a system input x_1 produces output y_1 , and a system input x_2 produces output y_2 , then a system input $x_1 + x_2$ produces an output $y_1 + y_2$.

2. Principle of Homogeneity (α is a constant)



That is, if a system input x_1 produces output y_1 , then a system input αx_1 produces an output αy_1 .

Examples



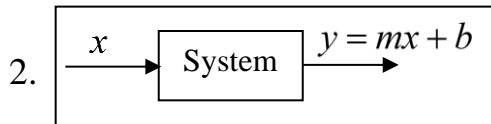
$$\alpha x \rightarrow m(\alpha x) = \alpha(mx) = \alpha y$$

... homogeneity satisfied

$$x_1 + x_2 \rightarrow m(x_1 + x_2) = mx_1 + mx_2 = y_1 + y_2$$

... superposition satisfied

System is **linear**.



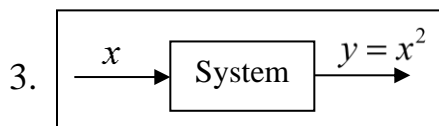
$$\alpha x \rightarrow m(\alpha x) + b = \alpha(mx) + b \neq \alpha y$$

... homogeneity **not** satisfied

$$x_1 + x_2 \rightarrow m(x_1 + x_2) + b = mx_1 + mx_2 + b \neq y_1 + y_2$$

... superposition **not** satisfied

System is **nonlinear**.



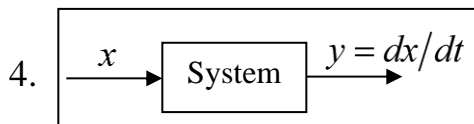
$$\alpha x \rightarrow (\alpha x)^2 = \alpha^2 x^2 = \alpha^2 y \neq \alpha y$$

... homogeneity **not** satisfied

$$x_1 + x_2 \rightarrow (x_1 + x_2)^2 = x_1^2 + x_2^2 + 2x_1x_2 \neq y_1 + y_2$$

... superposition **not** satisfied

System is **nonlinear**.



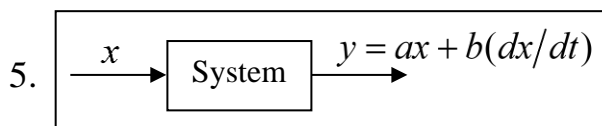
$$\alpha x \rightarrow d(\alpha x)/dt = \alpha(dx/dt) = \alpha y$$

... homogeneity satisfied

$$x_1 + x_2 \rightarrow d(x_1 + x_2)/dt = (dx_1/dt) + (dx_2/dt) = y_1 + y_2$$

... superposition satisfied

System is **linear**.



$$\alpha x \rightarrow a(\alpha x) + b(d(\alpha x)/dt) = \alpha(ax + b(dx/dt)) = \alpha y$$

... homogeneity satisfied

$$x_1 + x_2 \rightarrow a(x_1 + x_2) + b(d(x_1 + x_2)/dt)$$

$$= (ax_1 + b(dx_1/dt)) + (ax_2 + b(dx_2/dt)) = y_1 + y_2$$

... superposition satisfied

System is **linear**.