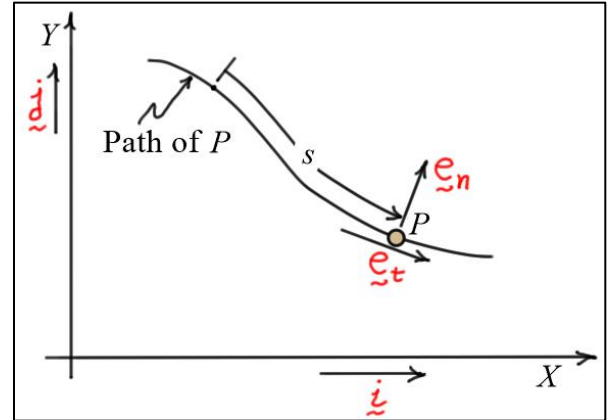


Elementary Dynamics

Curvilinear Motion – Normal and Tangential Components

Normal and Tangential Components

Normal and tangential components refer to components that are *normal* and *tangential* to the path of P . These directions are defined by the unit vectors \underline{e}_n and \underline{e}_t , respectively. Note that they are different from the unit vectors \underline{i} and \underline{j} in that their *orientation changes with time*.



Using normal and tangential components, the *velocity* of P can be written as

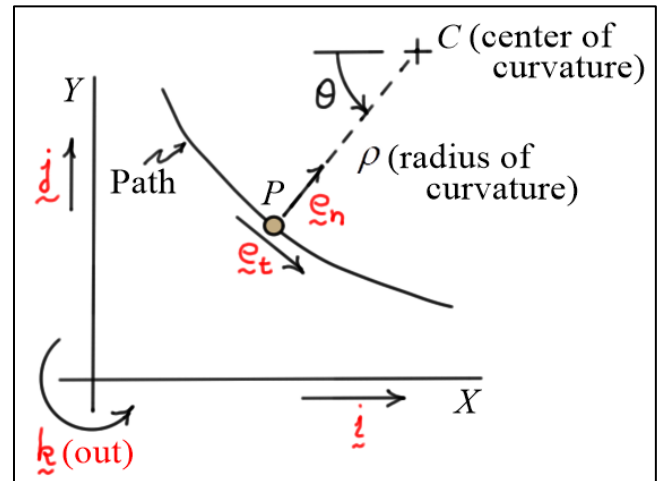
$$\underline{v} = v \underline{e}_t = \frac{ds}{dt} \underline{e}_t = \dot{s} \underline{e}_t.$$

The *acceleration* of P is found by differentiating the velocity with respect to time (using the product rule)

$$\underline{a} = \dot{v} \underline{e}_t + v \dot{\underline{e}}_t = \dot{v} \underline{e}_t + v (\dot{\theta} \underline{k} \times \underline{e}_t) = \dot{v} \underline{e}_t + v \dot{\theta} \underline{e}_n$$

From calculus we know that \dot{s} can be related to $\dot{\theta}$ using ρ the radius of curvature. Specifically,

$$v = \rho \dot{\theta} \quad \text{or} \quad \dot{\theta} = v / \rho$$



Substituting this result into the acceleration gives the final result

$$\underline{a} = \dot{v} \underline{e}_t + \left(\frac{v^2}{\rho} \right) \underline{e}_n \quad \left\{ \begin{array}{l} a_t = \dot{v} = \frac{dv}{dt} \text{ (tangential acceleration)} \\ a_n = \frac{v^2}{\rho} \text{ (normal acceleration)} \end{array} \right.$$

Special Case: Circular Motion

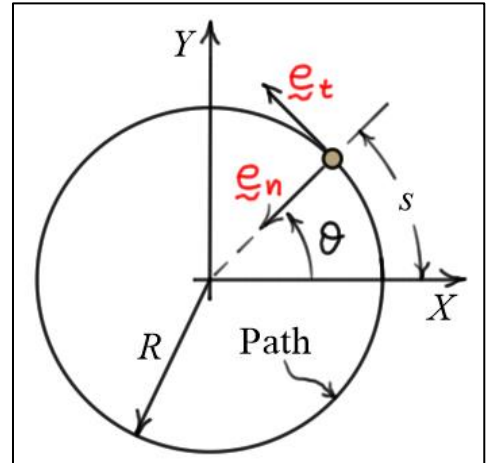
In the special case of *circular motion*, we have

$$s = R\theta$$

where θ is measured in radians. *Differentiating* with respect to time gives

$$\dot{s} = R\dot{\theta} = R\omega \quad \text{and} \quad \ddot{s} = R\ddot{\theta} = R\alpha.$$

Substituting these results into the *velocity* and *acceleration* formulas from above gives



$$\underline{v} = \dot{s} \underline{e}_t = R\dot{\theta} \underline{e}_t$$

and

$$\underline{a} = \ddot{s} \underline{e}_t + \left(\frac{\dot{s}^2}{R} \right) \underline{e}_n = R\ddot{\theta} \underline{e}_t + R\dot{\theta}^2 \underline{e}_n = R\alpha \underline{e}_t + R\omega^2 \underline{e}_n$$