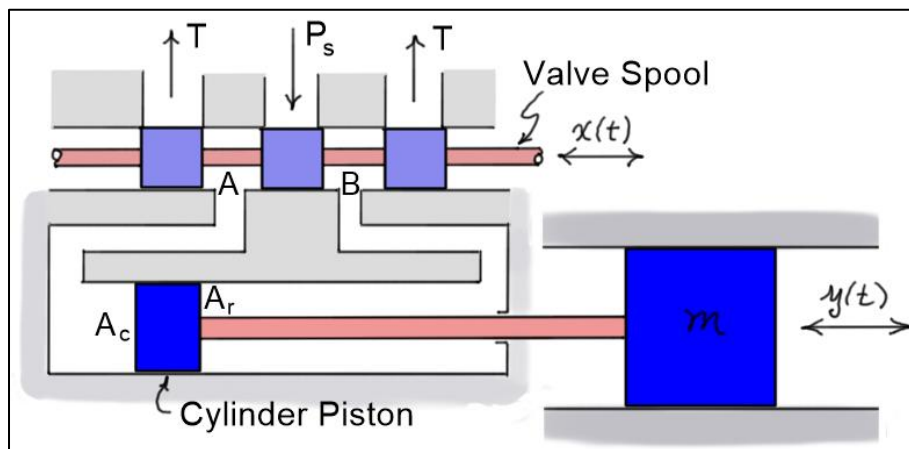


Introductory Control Systems Hydraulic Positioning System

Positioning System – Definition of Terms

- Incompressible fluid
- A_c = cap end piston area
- A_r = rod end piston area
- m = mass of load
- b = damping coefficient
- P_s = constant supply pressure
- P = pressure on the piston
- $p = \Delta P$, the change in P
- X = valve spool position
- $x = \Delta X$, the change in X
- Y = load position
- $y = \Delta Y$, the change in Y



Operation

- If $X > 0$, the **pressure source** is applied to the **A** port of the valve and the **cap end** of the cylinder causing the load to **move right**. Return flow to the tank is through the **B** port.
- If $X < 0$, the **pressure source** is applied to the **B** port of the valve and the **rod end** of the cylinder causing the load to **move left**. Return flow to the tank is through the **A** port.

Flow Model

If $X > 0$, the pressure source is applied to the **A** port of the valve. As a result, fluid flows into the piston chamber. The **volumetric flow rate** Q through the valve is a function of the spool position X and the pressure P in the piston chamber.

$$Q = g(X, P) \quad (1)$$

To simplify the model, Eq. (1) can be **linearized** about some operational (set) point (X_0, P_0) .

This is done using a **Taylor series expansion** as discussed in earlier notes. The **change in flow rate** can be written as

$$q \triangleq \Delta Q = \left(\frac{\partial g}{\partial X} \right)_{X_0, P_0} \Delta X + \left(\frac{\partial g}{\partial P} \right)_{X_0, P_0} \Delta P$$

$$= (k_x)x - (k_p)p$$
(2)

where k_x and k_p represent the *derivatives* of the function $g(X, P)$ with respect to X and P , respectively. The minus sign in the second of equations (2) indicates the flow rate *decreases* as the *pressure* in the piston chamber *increases*.

Assuming the fluid is *incompressible*, the *volumetric flow rate* can be related to the *speed* of the piston as follows.

$$Q = A_c \dot{Y}$$
(3)

Letting $Q = Q_0 + q$, $\dot{Y} = \dot{Y}_0 + \dot{y}$, and $Q_0 = A_c \dot{Y}_0$, then *changes* in the *volumetric flow rate* can be related to *changes* in the *speed* of the piston as follows.

$$q = A_c \dot{y}$$
(4)

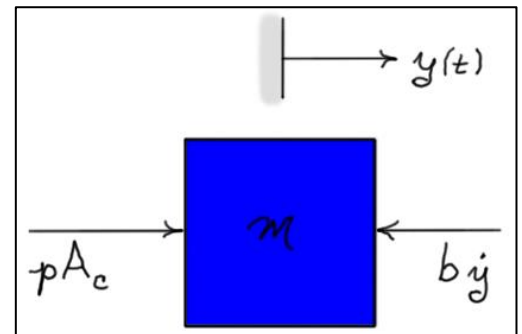
Combining Eqs. (4) and (2) gives a relationship between the changes in pressure, valve spool position, and speed of the piston.

$$p = (k_x x - A_c \dot{y}) / k_p$$
(5)

Model for Piston Movement

Assuming the pressure on the rod end of the piston is small compared to the pressure on the cap end, Newton's law gives

$$\rightarrow \sum_+ F = p A_c - b \dot{y} = m \ddot{y}$$



Note that it is assumed here that the *nominal velocity* is *constant*, and the *nominal pressure* and *damping forces* cancel from the force summation. Hence, the force summation represents changes from the nominal, constant velocity condition.

Rearranging the equation, and substituting for the pressure from Eq. (5) gives

$$\boxed{m\ddot{y} + \left(b + \frac{A_c^2}{k_p}\right)\dot{y} = A_c \left(\frac{k_x}{k_p}\right)x} \quad (X > 0) \quad (6)$$

If $X < 0$, then $p = (A_r \dot{y} - k_x x) / k_p$ and $\rightarrow \sum_+ F = -p A_r - b \dot{y} = m \ddot{y}$. In this case, the model equation is

$$\boxed{m\ddot{y} + \left(b + \frac{A_r^2}{k_p}\right)\dot{y} = A_r \left(\frac{k_x}{k_p}\right)x} \quad (X < 0) \quad (7)$$

Note here that x and y are still measured *positive* to the *right*.

Notes:

- Because the piston areas A_c and A_r are not equal, Eqs. (6) and (7) represent *two different* dynamic responses. The hydraulic cylinder will respond differently in extension and retraction.
- Conversely, if the cylinder is a *double-rod cylinder* with $A_c = A_r$, then the same model applies in both directions. Extension and retraction dynamics will be identical.
- The motions described by Eqs. (6) and (7) are *second-order, over-damped* responses.
- If the mass of the load is small ($m \approx 0$), then the response is *first order*.