

Introductory Motion and Control

Armature Controlled DC Motor Transfer Functions

(Reference: Dorf and Bishop, Modern Control Systems, 10th Ed., Pearson/Prentice-Hall, Inc. 2005)

In an *armature-current controlled DC motor*, the field current i_f is held constant, and the armature current is controlled through the armature voltage V_a . In this case, the *motor torque increases linearly* with the *armature current*.

$$T_m = K_{ma} i_a \quad (1)$$

The coefficient K_{ma} is a *constant* that depends on the chosen motor. The *transfer function* from the *input armature current* to the *resulting motor torque* is

$$\frac{T_m(s)}{I_a(s)} = K_{ma} \quad (2)$$

The *voltage-current relationship* for the armature side of the motor is

$$V_a = V_R + V_L + V_b = R_a i_a + L_a (di_a/dt) + V_b \quad (3)$$

V_b represents the “*back EMF*” induced by the *rotation* of the *armature windings* in a *magnetic field*. V_b is *proportional* to the *rotational speed* ω , that is, $V_b(s) = K_b \omega(s)$.

Taking Laplace transforms of Eq. (3) gives

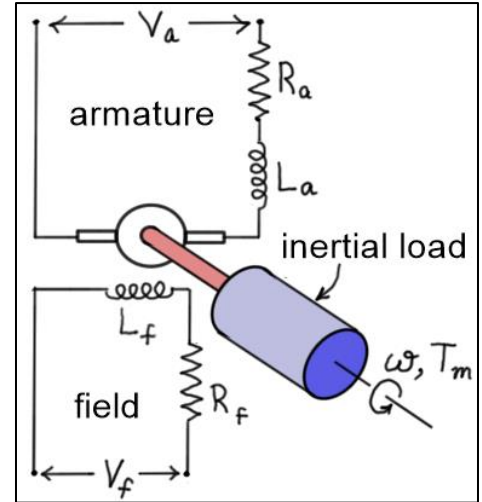
$$V_a(s) - V_b(s) = (R_a + L_a s) I_a(s) \quad \text{or} \quad V_a(s) - K_b \omega(s) = (R_a + L_a s) I_a(s) \quad (4)$$

Applying *Newton’s second law* (by summing moments) for the rotational motion of the motor gives

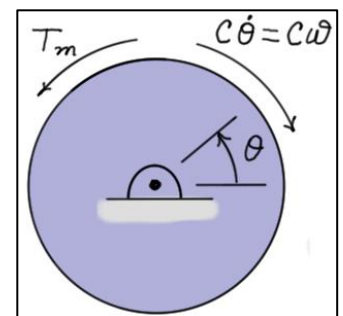
$$\sum M = T_m - c\omega = J\dot{\omega} \quad (\text{CCW positive})$$

or

$$J\dot{\omega} + c\omega = T_m \quad (5)$$



DC Motor

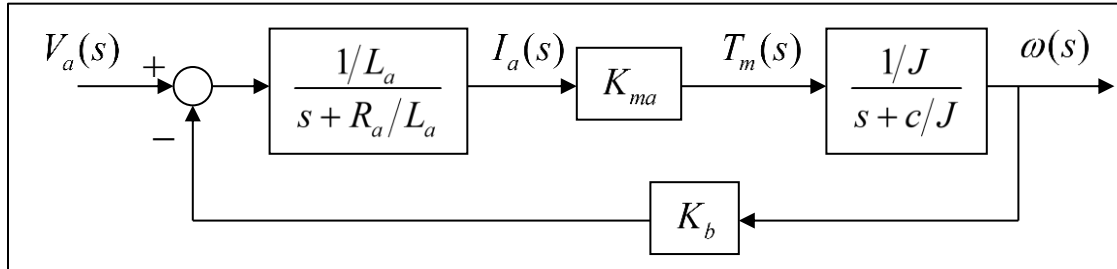


Free Body Diagram of the Inertial Load

Taking Laplace transforms of both sides of Eq. (5) gives the **transfer function** from the **input motor torque to rotational speed changes**.

$$\boxed{\frac{\omega}{T_m}(s) = \frac{(1/J)}{s + (c/J)}} \quad (1^{\text{st}} \text{ order system}) \quad (6)$$

Together, Eqs. (2), (4) and (6) can be represented by the following **closed-loop block diagram**.



Using **block diagram reduction**, the transfer function from the **input armature voltage to the resulting speed change** is found to be

$$\boxed{\frac{\omega}{V_a}(s) = \frac{(K_{ma}/L_a J)}{(s + R_a/L_a)(s + c/J) + (K_b K_{ma}/L_a J)}} \quad (2^{\text{nd}} \text{ order system}) \quad (7)$$

If the **time constant** of the **electrical circuit** is **small** compared to the **time constant** of the **load dynamics**, the transfer function of Eq. (7) may be approximated by the following first-order transfer function.

$$\boxed{\frac{\omega}{V_a}(s) = \frac{K_{ma} / R_a J}{s + (cR_a + K_b K_{ma}) / R_a J}} \quad (1^{\text{st}} \text{ order system}) \quad (8)$$

The transfer function from the **input armature voltage** to the resulting **angular position** change is found by multiplying Eqs. (7) and (8) by $1/s$.

$$\boxed{\frac{\theta}{V_a}(s) = \frac{K_{ma} / R_a J}{s(s + (cR_a + K_b K_{ma}) / R_a J)}} \quad (2^{\text{nd}} \text{ order system}) \quad (9)$$

Note this transfer function also represents a second-order differential equation with inertia and damping, but no stiffness (same form as for a hydraulic cylinder!).