Multibody Dynamics
Conversion of Direction Cosines to 1-2-3 Body-Fixed Angle Sequence

Given the coordinate transformation matrix $[R]$, the ***orientation angles*** for a 1-2-3 body-fixed angle sequence may be computed as follows. First, recall that $[R]$ may be written as

$$\boxed{[R]=\begin{bmatrix} C_2C_3 & C_1S_3+S_1S_2C_3 & S_1S_3-C_1S_2C_3 \\ -C_2S_3 & C_1C_3-S_1S_2S_3 & S_1C_3+C_1S_2S_3 \\ S_2 & -S_1C_2 & C_1C_2 \end{bmatrix}}$$

The three orientation angles may then be calculated by observation

$$\boxed{\theta_2=\sin^{-1}(R_{31})} \qquad (1)$$

$$\boxed{\theta_1=\tan^{-1}\left(\frac{-R_{32}}{R_{33}}\right)} \qquad (2)$$

$$\boxed{\theta_3=\tan^{-1}\left(\frac{-R_{21}}{R_{11}}\right)} \qquad (3)$$

Note that Eqs. (2) and (3) are singular when $\cos(\theta_2)=0$, that is, when $\theta_2=\frac{\pi}{2}$. In this case,

$$R_{12}=C_1S_3+S_1C_3=\sin(\theta_1+\theta_3) \qquad (4)$$

$$R_{22}=C_1C_3-S_1S_3=\cos(\theta_1+\theta_3) \qquad (5)$$

So, when $\theta_2=\frac{\pi}{2}$, Eqs. (4) and (5) can be used to ***solve for the sum*** of the other two angles. Using Eqs. (4) and (5), write

$$\boxed{\theta_1+\theta_3=\tan^{-1}\left(\frac{R_{12}}{R_{22}}\right)}$$