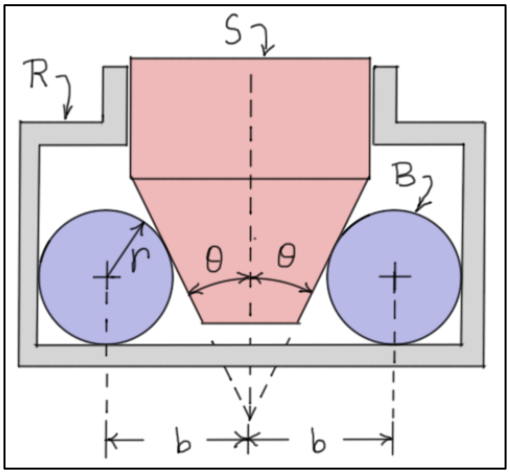
Intermediate Dynamics: Thrust Bearing Example

(Reference: Kane and Levinson, *Dynamics: Theory and Applications*, McGraw-Hill, 1985.

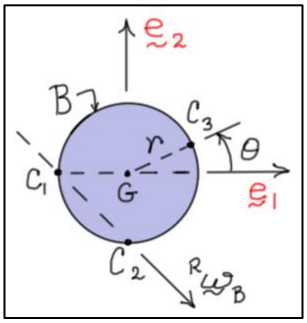
Problem:

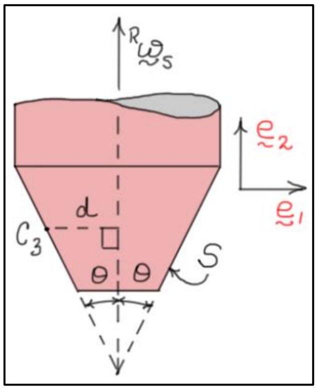
Given the thrust bearing in the diagram, show that for ***pure rolling*** between the shaft S and the bearing B, it is required that $\boxed{b=\dfrac{r(1+S_\theta)}{C_\theta - S_\theta}}$. Note that ***pure rolling*** occurs between S and B if ***no slipping*** occurs and if ${}^B\underset{\sim}{\omega}_S$ the angular velocity of the shaft (S) relative to the bearing (B) is ***parallel*** to the ***common tangent plane*** between S and B. Assume also that ***no slip*** occurs between the bearing and the race (R).



Solution:

1. Consider the separate diagrams below of the shaft and the bearing. Points C_1 and C_2 represent the contact points between the bearing and the race (R), and C_3 represents the contact point between the shaft and the bearing. The unit vectors $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$ represent the horizontal and vertical directions in the plane defined by these three points. To complete the unit vector set, a third unit vector is defined as $\underset{\sim}{e}_3 = \underset{\sim}{e}_1 \times \underset{\sim}{e}_2$.





Given this set-up and the ***no-slip conditions*** at points C_1, C_2, and C_3, the angular velocities of the shaft and the bearing may be written as

$$\boxed{{}^R\underset{\sim}{\omega}_S = {}^R\omega_S\,\underset{\sim}{e}_2} \qquad \boxed{{}^R\underset{\sim}{\omega}_B = {}^R\omega_B\left(\tfrac{\sqrt{2}}{2}\underset{\sim}{e}_1 - \tfrac{\sqrt{2}}{2}\underset{\sim}{e}_2\right)} \qquad (1)$$

2. The ***angular velocities*** of the shaft and the bearing in R can be ***related*** by calculating the velocity of the contact point C_3. In this process, advantage is taken of the fact that the velocities of the points C_1 and C_2 relative to R are zero due to the ***no-slip*** condition.

$$
\begin{aligned}
{}^R\underset{\sim}{v}_{C_3} &= {}^R\underset{\sim}{v}_{C_1} + {}^R\underset{\sim}{v}_{C_3/C_1} = 0 + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{r}_{C_3/C_1}\right) \\
&= {}^R\omega_B\left(\tfrac{\sqrt{2}}{2}\underset{\sim}{e}_1 - \tfrac{\sqrt{2}}{2}\underset{\sim}{e}_2\right)\times\left(r(1+C_\theta)\underset{\sim}{e}_1 + rS_\theta\underset{\sim}{e}_2\right) \qquad (C_3 \text{ is a point on the } \textbf{\textit{bearing}})
\end{aligned}
$$

$$
\Rightarrow \boxed{{}^R\underset{\sim}{v}_{C_3} = \tfrac{\sqrt{2}}{2} r\left(S_\theta + (1+C_\theta)\right) {}^R\omega_B\, \underset{\sim}{e}_3}
$$

Also,

$$
\boxed{{}^R\underset{\sim}{v}_{C_3} = d\, {}^R\omega_S\, \underset{\sim}{e}_3 = \left(b - r\,C_\theta\right) {}^R\omega_S\, \underset{\sim}{e}_3} \qquad (C_3 \text{ is a point on the } \textbf{\textit{shaft}})
$$

Assuming there is ***no-slip*** between the ***bearing*** and the ***shaft***, these two velocities must be equal. Setting the two equal leads to the conclusion that

$$
\boxed{{}^R\omega_S = \tfrac{\sqrt{2}}{2}\left[\frac{r\left(1+S_\theta+C_\theta\right)}{\left(b-rC_\theta\right)}\right]{}^R\omega_B} \tag{2}
$$

3. The ***angular velocities*** of the shaft and the bearing can also be related using the ***summation rule*** for angular velocities. That is,

$$
\boxed{{}^R\underset{\sim}{\omega}_S = {}^R\underset{\sim}{\omega}_B + {}^B\underset{\sim}{\omega}_S} \tag{3}
$$

Here, ${}^R\underset{\sim}{\omega}_B$ the angular velocity of the bearing is given by Eq. (1), ${}^R\underset{\sim}{\omega}_S$ the angular velocity of the shaft is given by Eqs. (1) and (2), and for ***pure rolling*** between S and B,

$$
\boxed{{}^B\underset{\sim}{\omega}_S = {}^B\omega_S\left(-S_\theta\underset{\sim}{e}_1 + C_\theta\underset{\sim}{e}_2\right)}. \tag{4}
$$

Substituting from Eqs. (1), (2), and (4) into Eq. (3) gives the following two scalar equations.

$$
\boxed{\begin{aligned}
&\tfrac{\sqrt{2}}{2}\,{}^R\omega_B - S_\theta\,{}^B\omega_S = 0 \\
&-\tfrac{\sqrt{2}}{2}\,{}^R\omega_B + C_\theta\,{}^B\omega_S = \tfrac{\sqrt{2}}{2}\left[\frac{r\left(1+S_\theta+C_\theta\right)}{\left(b-rC_\theta\right)}\right]{}^R\omega_B
\end{aligned}} \tag{5}
$$

Multiplying the first equation by C_θ and the second by S_θ, adding the two equations, and simplifying gives the desired result.

$$
\boxed{b = \frac{r\left(1+S_\theta\right)}{C_\theta - S_\theta}}
$$