Multibody Dynamics
Generalized Coordinates, Quasi-Coordinates, and Generalized Speeds

Recall that ***generalized coordinates*** are a set of geometric variables related to the ***physical coordinates*** used to define the ***degrees of freedom*** of the system. For a ***rigid body*** in three dimensions, we have ***six degrees of freedom***, three of translation and three of rotation. The three degrees of freedom of translation can be defined as the Cartesian coordinates (x_1, x_2, x_3) of some point O of the body. The three degrees of freedom of rotation can be defined by a set of three orientation angles.

If we define $\{q\}$ as the vector of ***generalized coordinates*** of the body, then we have

$$\{q\} = \begin{bmatrix} x_1 & x_2 & x_3 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^T$$

We can define a set of ***generalized speeds*** as the time derivatives of the generalized coordinates.

$$\{\dot{q}\} = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 \end{bmatrix}^T$$

However, generalized speeds may also be defined as ***linear combinations*** of the ***time derivatives*** of the generalized coordinates. For example, ***angular velocity components*** of a body are linear combinations of the derivatives of the orientation angles, but they are not the time derivatives of any coordinates. The generalized coordinates that correspond to the angular velocity components are called ***quasi-coordinates*** (because they do not exist). Using this idea, we can define $\{u\}$ a set of ***generalized speeds*** for a rigid body to be

$$\{u\} = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$$

Note: For more detailed and general discussions of generalized coordinates, see the following references.

1. L. Meirovitch, *Methods of Analytical Dynamics*, McGraw-Hill, 1970
2. T.R. Kane and D.A. Levinson, *Dynamics: Theory and Applications*, McGraw-Hill, 1985
3. H. Baruh, *Analytical Dynamics*, WCB/McGraw-Hill, 1999
4. H. Josephs and R.L. Huston, *Dynamics of Mechanical Systems*, CRC Press, 2002