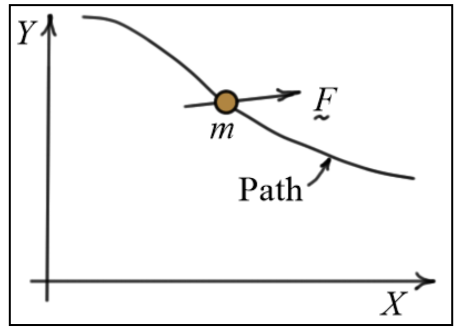
Elementary Dynamics
Principle of Impulse and Momentum for Particles

Recall that Newton's second law for a particle can be written as $\underset{\sim}{F} = m\underset{\sim}{a}$, where $\underset{\sim}{F}$ is the ***resultant*** force acting on the particle and $\underset{\sim}{a}$ is its ***acceleration***. Recall also that the force and the acceleration are generally not tangent to the path, except in the case of rectilinear motion.



Previously, we developed an ***integrated scalar form*** of Newton's second law, namely the ***principle of work and energy***. Here we develop an ***integrated vector form*** of Newton's second law called the ***principle of impulse and momentum***. Integrating Newton's law with respect to ***time*** gives:

$$\underset{\sim}{I}_{1\to 2} \triangleq \int_{t_1}^{t_2} \underset{\sim}{F}\, dt = m\int_{t_1}^{t_2} \left(\frac{d\underset{\sim}{v}}{dt}\right) dt = m\underset{\sim}{v}_2 - m\underset{\sim}{v}_1 = \Delta\underset{\sim}{L} \qquad \text{or} \qquad \boxed{m\underset{\sim}{v}_1 + \int_{t_1}^{t_2} \underset{\sim}{F}\, dt = m\underset{\sim}{v}_2}$$

This is the ***principle of linear impulse and momentum***. It states that the ***resultant linear impulse*** on the system over the ***time interval*** from $t_1 \to t_2$ gives rise to a ***change*** in ***linear momentum***. Note that this equation, like Newton's law, is a ***vector*** equation. Also, note that the term "***linear***" means "***translational***" as opposed to "***angular***" or "***rotational***".

This principle can be extended to a system of particles by noting that the ***total*** impulse of all equal and opposite internal forces in a system is ***zero***. So, in this case, for a system of particles, we have

$$\boxed{\sum m_i \left(\underset{\sim}{v}_i\right)_1 + \sum \int_{t_1}^{t_2} \underset{\sim}{F}_i\, dt = \sum m_i \left(\underset{\sim}{v}_i\right)_2} \qquad \text{or} \qquad \boxed{m\left(\underset{\sim}{v}_G\right)_1 + \sum \int_{t_1}^{t_2} \underset{\sim}{F}_i\, dt = m\left(\underset{\sim}{v}_G\right)_2}$$

Here, $\sum \int_{t_1}^{t_2} \underset{\sim}{F}_i\, dt$ represents the ***sum*** of the impulses of all ***external forces*** and ***internal forces*** acting on the system, and G represents the ***mass center*** of the system. The ***time-averaged, net force*** acting on the system is defined to be

$$\boxed{\underset{\sim}{F}_{avg} = \frac{1}{\Delta t}\sum\int_{t_1}^{t_2}\underset{\sim}{F}_i\,dt = \frac{1}{\Delta t}\underset{\sim}{I}_{1\to 2}}$$

Definitions: Forces that are large, act over a short interval of time, and cause ***significant*** changes in momentum are called ***impulsive*** forces. Forces that cause ***insignificant*** changes in momentum over short intervals of time are called ***non-impulsive*** forces. ***Weight forces*** are generally considered to be ***non-impulsive***.