Introductory Control Systems
Cramer's Rule for Solving a System of Linear Algebraic Equations

Consider a set of ***linear algebraic equations*** with known coefficient matrix $[A]$, known right-side vector $\{b\}$, and vector of unknowns $\{x\}$.

$$[A]\{x\}=\{b\}$$

One approach to solving these equations is ***Cramer's rule***. An ***illustration*** of how to apply Cramer's rule to a set of three equations follows. The ***extension*** of the rule to larger sets of equations should be obvious.

Given the set of three linear algebraic equations:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

Using Cramer's rule, the solutions for the three unknowns x_i $(i=1,2,3)$ can be written as follows.

$$x_1 = \frac{\det \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix}}{\det[A]} \qquad x_2 = \frac{\det \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}}{\det[A]} \qquad x_3 = \frac{\det \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{bmatrix}}{\det[A]}$$

Note the denominator of the solution for each of the unknowns is the same.