Elementary Dynamics
Impact of Particles

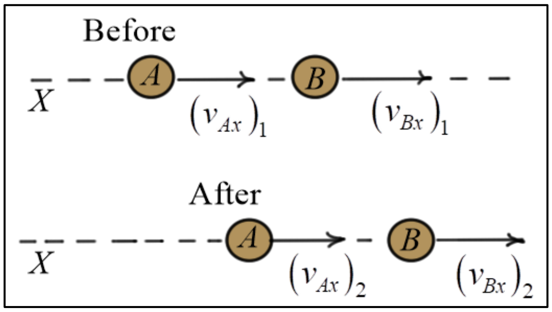
When two particles collide, the ***net impulse*** on the system (the two particles) is ***zero***, so the motion of the system must satisfy the principle of ***conservation*** of ***linear momentum***. That is,

$$\boxed{m_A\left(\underset{\sim}{v}_A\right)_1 + m_B\left(\underset{\sim}{v}_B\right)_1 = m_A\left(\underset{\sim}{v}_A\right)_2 + m_B\left(\underset{\sim}{v}_B\right)_2}$$

Note that this equation is a ***vector equation***, so it must be true in all directions. The ***velocity*** of the ***mass-center*** of the system is the same before and after the impact, and it is given by

$$\boxed{\underset{\sim}{v}_G = \left(m_A\underset{\sim}{v}_A + m_B\underset{\sim}{v}_B\right)/\left(m_A + m_B\right)}$$

Direct Central Impact



During ***direct central impact***, two particles are travelling along the same line at different speeds before and after the impact. The contact force between the particles is assumed to be along this same line as well. If the line is called the ***X-axis***, then the conservation of momentum states

$$\boxed{m_A\left(v_{Ax}\right)_1 + m_B\left(v_{Bx}\right)_1 = m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2} \qquad (1)$$

The relative velocities of the particles along the ***X-axis*** before and after impact can be related through the ***coefficient of restitution***, e, defined as

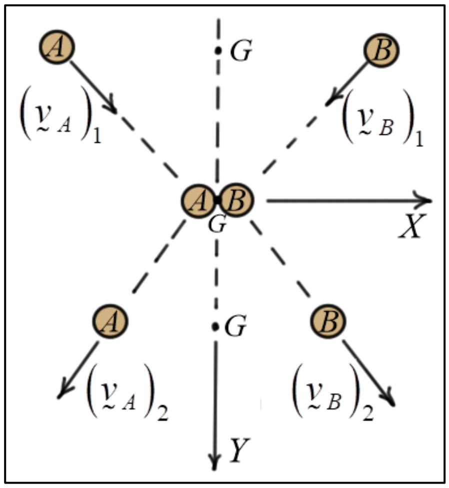
$$\boxed{e = \frac{\text{Restitution Impulse}}{\text{Deformation Impulse}} = \frac{\int R\,dt}{\int D\,dt} = \frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1}} \quad \text{or} \quad \boxed{e = \frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1}} \qquad (2)$$

The coefficient of restitution is a ***measure of how much energy is lost*** during the collision. The range of values is $0 \le e \le 1$. In a ***perfectly plastic*** collision $(e = 0)$, the particles remain together after the collision. In a ***perfectly elastic*** collision $(e = 1)$, no energy is lost, so the ***kinetic energy*** of the system is ***conserved***.

Equations (1) and (2) represent ***two equations*** that can be solved for ***two unknowns***. For example, if the initial velocities of the particles and the coefficient of restitution are known, then the final velocities can be calculated.

Oblique Central Impact

In the case of ***oblique central impact***, the two particles approach each other at some ***oblique angle***. The ***contact force*** between the two particles is assumed to be in the ***X-direction***, only. Because ***no forces*** are applied in the ***Y-direction***, velocities are ***conserved*** in that direction. That is, in the ***Y-direction***



$$\left(v_{Ay}\right)_2 = \left(v_{Ay}\right)_1 \quad \text{and} \quad \left(v_{By}\right)_2 = \left(v_{By}\right)_1$$

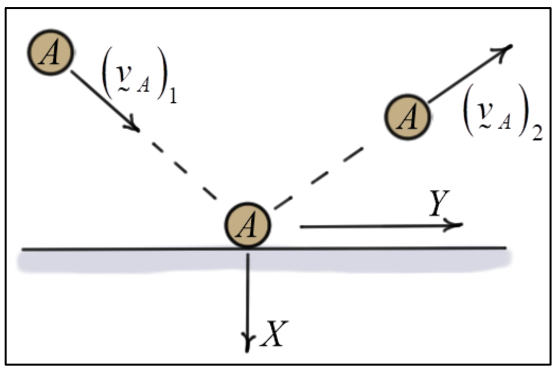
In the ***X-direction***, the conservation of linear momentum and coefficient of restitution equations apply.

$$m_A\left(v_{Ax}\right)_1 + m_B\left(v_{Bx}\right)_1 = m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2 \quad \text{and} \quad e = \frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1}$$

These equations represent ***four equations*** that can be solved for ***four unknowns***. For example, if the coefficient of restitution and all four of the initial velocity components are known, then these equations can be solved for the four final velocity components.

Collisions with a Fixed Surface

If a particle *A* strikes a fixed surface, then the ***linear momentum*** of *A* is ***not conserved***! If the contact force is assumed (by neglecting friction) to be only in the ***X-direction***, then the following equations apply:



$$\left(v_{Ay}\right)_2 = \left(v_{Ay}\right)_1 \quad \text{and} \quad \left(v_{Ax}\right)_2 = -e\left(v_{Ax}\right)_1$$

Note as a result of the above two equations that the ***angle of incidence*** is ***not equal to*** the ***angle of reflection*** unless the collision is perfectly elastic.