

Introductory Control Systems

Block Diagrams and Transfer Functions – Example

Problem:

The angular position of a rotating shaft is controlled with a torque actuator with a proportional-integral (PI) controller. The desired shaft angle is $\theta_d(t)$, the actual shaft angle is $\theta(t)$, the angle error is $e(t)$, the input voltage to the actuator is $v(t)$, and the actuator torque is $m(t)$. The equations that govern the individual processes are as follows.

$$\boxed{e(t) = \theta_d(t) - \theta(t)} \quad \boxed{v(t) = 2e(t) + 8 \int_0^t e(t) dt} \quad \boxed{\dot{m} + 3m = v(t)} \quad \boxed{\ddot{\theta} + 7\dot{\theta} = 4m(t)}$$

In the last two equations, the *over-dot* represents the *time derivative*.

Find:

- transfer functions $\frac{V}{E}(s)$, $\frac{M}{V}(s)$, and $\frac{\theta}{M}(s)$.
- Sketch the block diagram of the closed-loop system. Label all signals and transfer functions.
- Using block diagram reduction, find the closed-loop system transfer function $\frac{\theta}{\theta_d}(s)$.

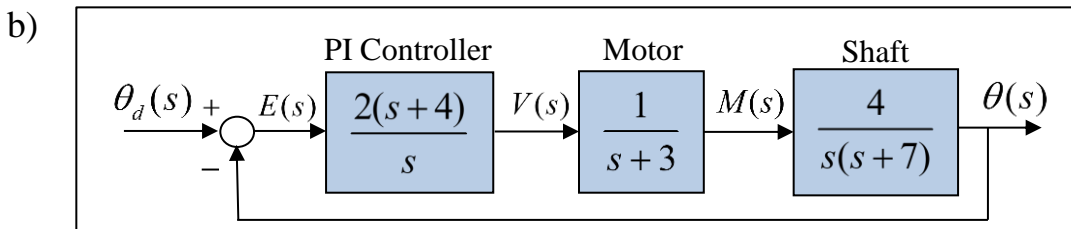
Solution:

- Applying Laplace transforms to the model equations gives

$$V(s) = 2E(s) + 8 \frac{E(s)}{s} = \left(2 + \frac{8}{s}\right)E(s) = \left(\frac{2(s+4)}{s}\right)E(s) \Rightarrow \boxed{\frac{V}{E}(s) = \frac{2(s+4)}{s}}$$

$$sM(s) + 3M(s) = V(s) \Rightarrow \boxed{\frac{M}{V}(s) = \frac{1}{s+3}}$$

$$s^2\theta(s) + 7s\theta(s) = 4M(s) \Rightarrow \boxed{\frac{\theta}{M}(s) = \frac{4}{s(s+7)}}$$



c) Using block diagram reduction, with $G = N_G/D_G$ and $H = 1$

$$\frac{\theta}{\theta_d}(s) = \frac{G}{1+GH} = \frac{N_G/D_G}{1+N_G/D_G} = \frac{N_G}{D_G + N_G} = \frac{8(s+4)}{s^2(s+3)(s+7)+8(s+4)}$$

$$\Rightarrow \boxed{\frac{\theta}{\theta_d}(s) = \frac{8(s+4)}{s^4 + 10s^3 + 21s^2 + 8s + 32}}$$