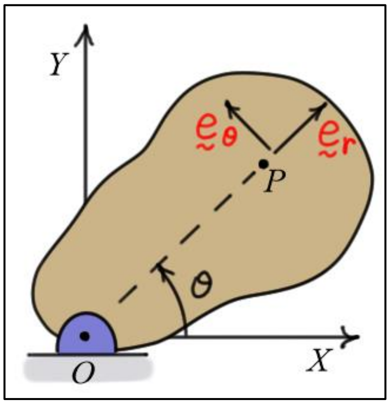
Elementary Dynamics
Rotational Motion about a Fixed Axis

A body B has ***pure rotational motion*** when one point (O) of the body is fixed, and all the other points of the body rotate around it. The body is rotating about an axis that passes through the fixed-point O and is perpendicular to the plane of motion. In this case, all the points of the body have ***circular motion***.



The kinematics of all points of B can be computed using the angular velocity and angular acceleration of B defined as follows.

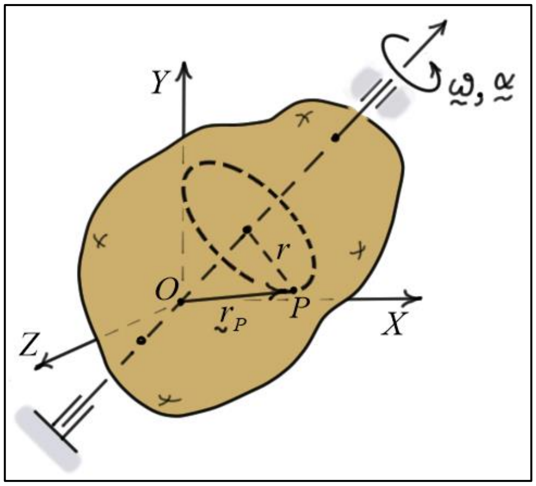
Angular Velocity: $\boxed{\underset{\sim}{\omega} = \frac{d\theta}{dt}\,\underset{\sim}{k} = \dot{\theta}\,\underset{\sim}{k}}$ Angular Acceleration: $\boxed{\underset{\sim}{\alpha} = \dot{\omega}\,\underset{\sim}{k} = \ddot{\theta}\,\underset{\sim}{k}}$

Here, θ is measured ***counterclockwise*** relative to a ***fixed horizontal line***. The velocity and acceleration of point P can be computed as

$$\underset{\sim}{v}_P = \frac{d}{dt}\left(\underset{\sim}{r}_P\right) = \frac{d}{dt}\left(r\,\underset{\sim}{e}_r\right) = r\,\dot{\underset{\sim}{e}}_r = r\left(\dot{\theta}\underset{\sim}{k} \times \underset{\sim}{e}_r\right) = \dot{\theta}\underset{\sim}{k} \times \left(r\underset{\sim}{e}_r\right) \quad \text{OR} \quad \boxed{\underset{\sim}{v}_P = \underset{\sim}{\omega} \times \underset{\sim}{r}_P = r\,\dot{\theta}\,\underset{\sim}{e}_\theta}$$

$$\boxed{\underset{\sim}{a}_P = \frac{d}{dt}\left(\underset{\sim}{\omega} \times \underset{\sim}{r}_P\right) = \left(\underset{\sim}{\alpha} \times \underset{\sim}{r}_P\right) + \underset{\sim}{\omega} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}_P\right) = -r\,\dot{\theta}^2\underset{\sim}{e}_r + r\,\ddot{\theta}\,\underset{\sim}{e}_\theta}$$

Usually, the velocity and acceleration are computed directly using the ***radial*** and ***transverse*** coordinates (r,θ) as shown in the boxed equations above. However, if the plane of motion is not easily identified, they may be calculated using vector expressions. For example, the velocity and acceleration of point P of the rotating three-dimensional shape can be calculated as



$$\boxed{\underset{\sim}{v}_P = \underset{\sim}{\omega} \times \underset{\sim}{r}_P} \quad \text{and} \quad \boxed{\underset{\sim}{a}_P = \left(\underset{\sim}{\alpha} \times \underset{\sim}{r}_P\right) + \underset{\sim}{\omega} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}_P\right)}$$

Here $\underset{\sim}{r}_P$ is the position vector of P relative to ***any*** point on the axis of rotation.