

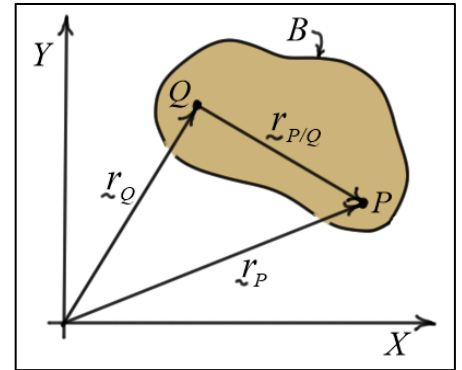
Elementary Dynamics

Relative Velocity of Two Points Fixed on a Rigid Body

The figure depicts a rigid body B moving in two dimensions. The two points P and Q are **fixed** on B . At any instant of time, the position vector of P can be written as

$$\underline{r}_P = \underline{r}_Q + \underline{r}_{P/Q}$$

Here, $\underline{r}_{P/Q}$ is called the position vector of P **relative** to Q .



The **velocities** of P and Q can be related to each other by **differentiating** the above equation.

$$\frac{d}{dt}(\underline{r}_P) = \frac{d}{dt}(\underline{r}_Q) + \frac{d}{dt}(\underline{r}_{P/Q}) \quad \text{or} \quad \underline{v}_P = \underline{v}_Q + \frac{d}{dt}(\underline{r}_{P/Q})$$

Note because the body is rigid, the position vector $\underline{r}_{P/Q}$ has **constant length**. However, the derivative of $\underline{r}_{P/Q}$ is **not zero**, because it will **change direction** as the body rotates.

To calculate the derivative of $\underline{r}_{P/Q}$, consider the figure at the right. Given \underline{e}_r is a unit vector pointed from Q towards P , the position vector $\underline{r}_{P/Q}$ can be written as

$$\underline{r}_{P/Q} = L \underline{e}_r$$

Here L represents the distance from Q to P . The derivative of $\underline{r}_{P/Q}$ can be calculated as follows.

$$\frac{d}{dt}(\underline{r}_{P/Q}) = \frac{d}{dt}(L \underline{e}_r) = L \dot{\underline{e}}_r = L(\dot{\theta} \underline{k}) \times \underline{e}_r = \dot{\theta} \underline{k} \times L \underline{e}_r = \underline{\omega} \times \underline{r}_{P/Q} = L \dot{\theta} \underline{e}_\theta$$

In this equation, $\underline{\omega} \triangleq \dot{\theta} \underline{k}$ is the **angular velocity** of the body. Combining this result with the boxed equation above gives

$$\underline{v}_P = \underline{v}_Q + (\underline{\omega} \times \underline{r}_{P/Q}) = \underline{v}_Q + \underline{v}_{P/Q} \quad \text{with} \quad \underline{v}_{P/Q} \triangleq \underline{\omega} \times \underline{r}_{P/Q}$$

This last equation defines $\underline{v}_{P/Q}$ the **velocity** of P **relative** to Q . This equation is used to relate the velocity of **two points fixed** on the **same rigid body**.

