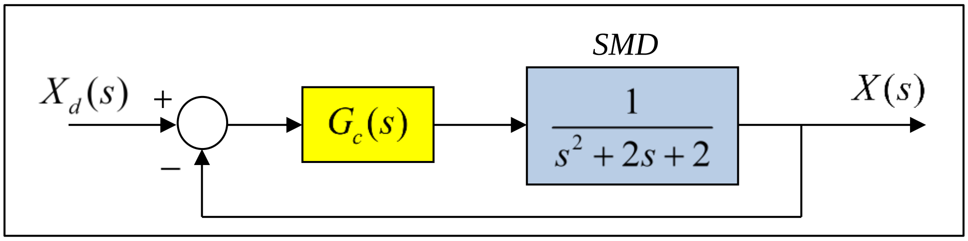
Introductory Motion and Control
Frequency Response Design of a Phase-Lead Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the ***frequency response design*** of a ***phase-lead compensator***, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the ***desired*** and ***actual positions*** of the mass.



Using proportional control ($G_c(s)=K$), ***large gains*** are required to control steady-state error of a step response. Unfortunately, large gains produce ***undesirable***, ***oscillatory*** closed-loop response. Below, a ***phase-lead*** compensator is designed to control the steady-state error and give desirable transient response.

Problem: Design a phase-lead compensator so the closed-loop system has a ***steady-state position error*** $e_{ss}=1-x_{ss}<0.05$ to a unit step input and a ***phase margin*** $PM=45\text{ (deg)}$. Plot the step response of the resulting closed-loop system.

Frequency Response Design

Step 1: Determine the ***required compensator gain*** to satisfy the ***error specification.***

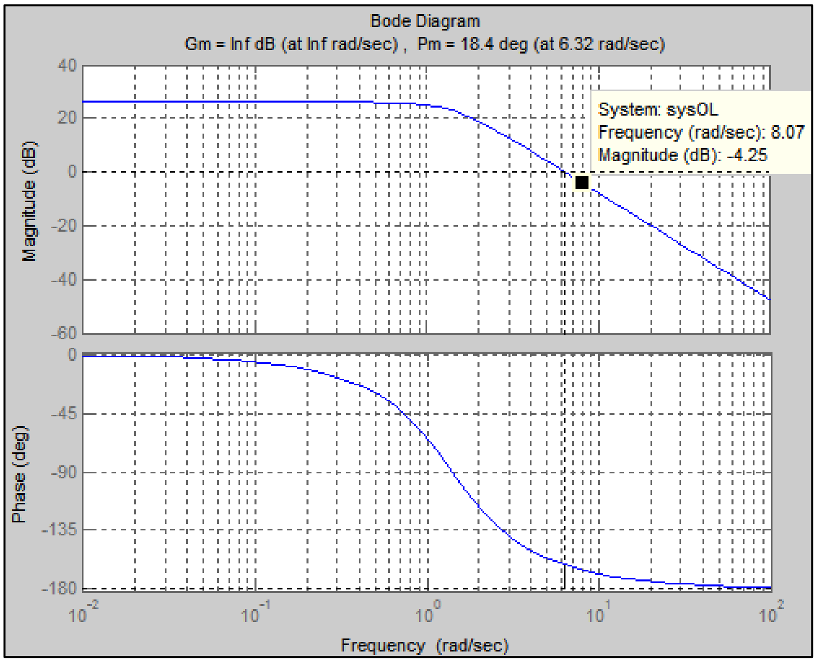
The ***steady state error*** can be defined in terms of the loop transfer function as

$$e_{ss}=\lim_{s\to 0}\left[\frac{1}{1+GH(s)}\right]=\frac{1}{1+K/2}=\frac{1}{1+K_p}<0.05$$

To satisfy this specification, $K_p>19$ and $K>38$. $K=40$ is used below as a starting value.

Step 2: Evaluate the ***phase margin*** of the ***uncompensated system.***

Using MATLAB, the phase margin of this (the ***uncompensated*** system) is $PM\cong 18.4\text{ (deg)}$. This is obviously ***well below*** the ***desired*** phase margin. See plot below.



Phase Margin of the Uncompensated System is 18.4 (deg)

Step 3: Calculate the ratio $\alpha = p/z$

An ***additional*** 27 degrees of ***phase margin*** are required, so the required value of α is

$$\alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)} = \frac{1+\sin(27)}{1-\sin(27)} = 2.663$$

Step 4: Find ω_m on the Bode diagram of the uncompensated system

The magnitude $-10\log(\alpha) = -4.254$ (db) occurs at approximately 8 (rad/sec). (See plot above.) So, set $\omega_m \cong 8$ (rad/sec), $p = \omega_m\sqrt{\alpha} \approx 13.06$, and $z = p/\alpha \approx 4.9$. The resulting compensator after the first iteration is

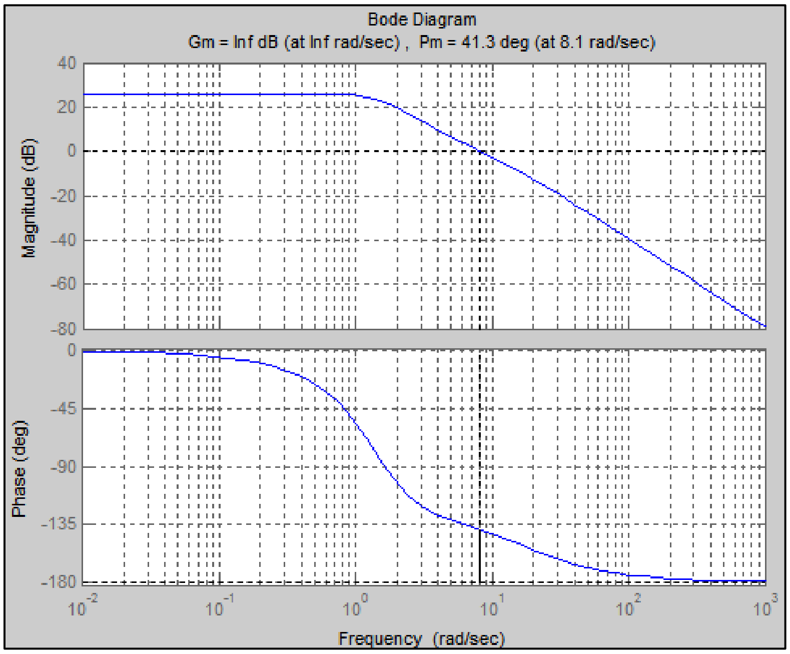
$$G_c(s) = 2.663\left[\frac{s+4.9}{s+13.06}\right]$$ (phase-lead compensator)

Step 5: Check the ***phase margin*** of the ***new compensated system.***

The Bode diagram of the loop transfer function of the compensated system with

$$GH(s) = 2.663\left[\frac{s+4.9}{s+13.06}\right]\left[\frac{40}{s^2+2s+2}\right]$$

shows that the phase margin is $PM = 41.3$ (deg). See plot below.



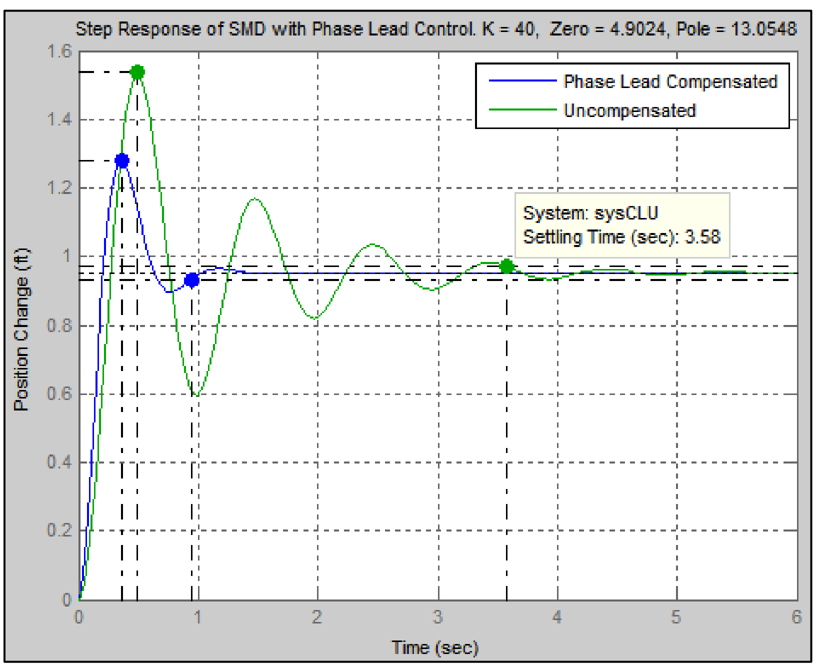
Phase Margin of Compensated System is 41.3 (deg)

Step 6: Repeat steps 3-5 until the desired phase margin is obtained.

If a larger phase margin is desired, steps 3-5 above can be repeated using a desired additional phase margin of higher than 27 degrees. For this example, the above values are used below to check the step response.

Step 7: Check the step response.

The step response of the uncompensated system shows a large overshoot (over 60%) and low damping (settling time of 3.58 (sec)), while the step response of the compensated system shows a smaller overshoot (around 35%) with higher damping (settling time of 0.94 (sec)).



Step response of the compensated system is much better than the uncompensated system. ***Try increasing the phase margin to get a better response.***

Settling times:
Uncompensated = 3.58 (sec)
Compensated = 0.94 (sec)

Frequency Response of Compensated Closed Loop System:

The frequency response of the closed loop system is shown below. The ***bandwidth*** of the closed loop system is approximately 13 (rad/s).