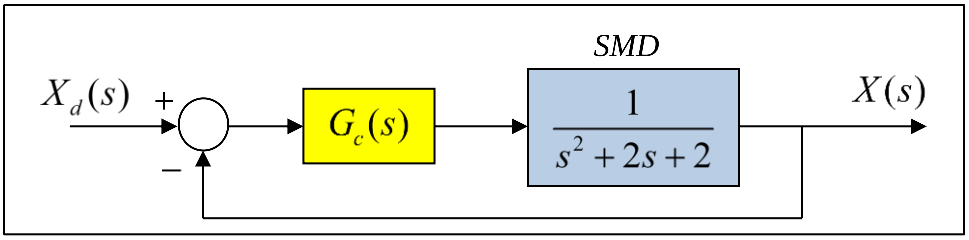
Introductory Motion and Control
Root Locus Design of a Phase-Lead Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the ***root locus design of a phase-lead compensator***, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the desired and actual positions of the mass.



Using proportional control ($G_c(s)=K$), ***large gains*** are required to control ***steady-state error*** to a step input. Unfortunately, large gains produce ***undesirable***, ***oscillatory*** closed-loop response. A phase-lead compensator is designed below to ***control*** the ***steady-state error*** and give ***desirable transient response***.

Problem: Design a phase-lead compensator so the closed-loop system has a settling time $T_s < 1\ (\text{sec})$, a damping ratio of the complex roots $\zeta > 0.5$, and a small steady-state position error. Plot the step response of the resulting closed-loop system.

Root Locus Design:

Step 1: Examine ***RL diagram*** of ***uncompensated system***. (same as proportional control)

The root locus of the ***uncompensated*** system is ***very simple***. The poles of $GH(s)$ are $-1 \pm 1j$. For $K>0$ the roots move to infinity along the asymptotes at $\sigma_A = -1$. For $K<0$ the roots move into the break point at $s=-1$ and then move along the positive and negative real axis.

Step 2: ***Evaluate*** how the compensator ***changes the RL diagram***.

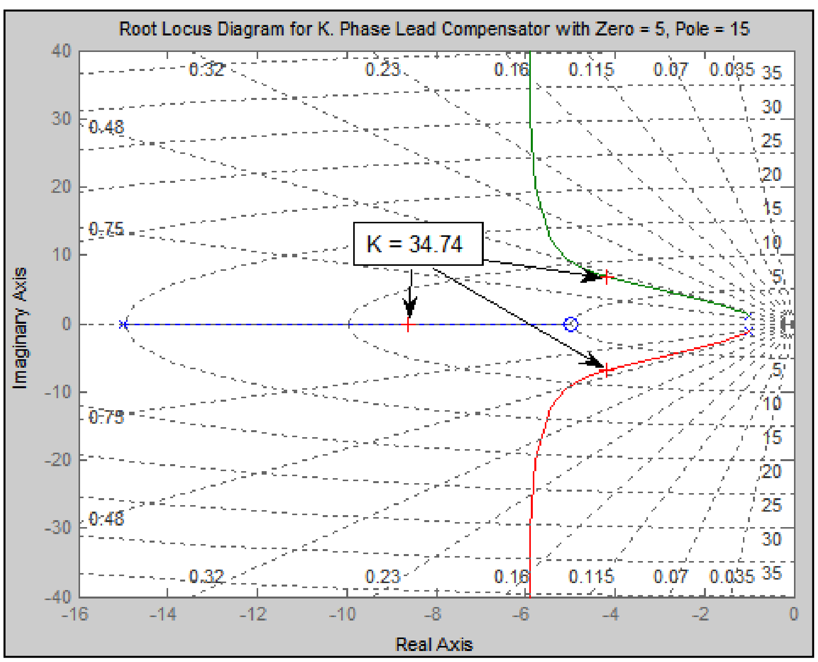
The loop transfer function of the compensated system is $GH(s) = \dfrac{K(s+z)}{(s+p)(s^2+2s+2)}$, so the system has asymptotes at $\phi_A = \pm 90\ (\text{deg})$ that intersect the real axis at $\sigma_A = \dfrac{2(-1)-p+z}{2}$. ***To ensure a settling time of less than 1 second***, $\sigma_A < -4$, or a pole-zero separation of $z-p<-6$.

As a first design, ***assume that*** $z - p = -10$. Note that this is only a starting point. More separation may be necessary.

Step 3: Try ***different pole-zero combinations*** to see effect on ***RL diagram***

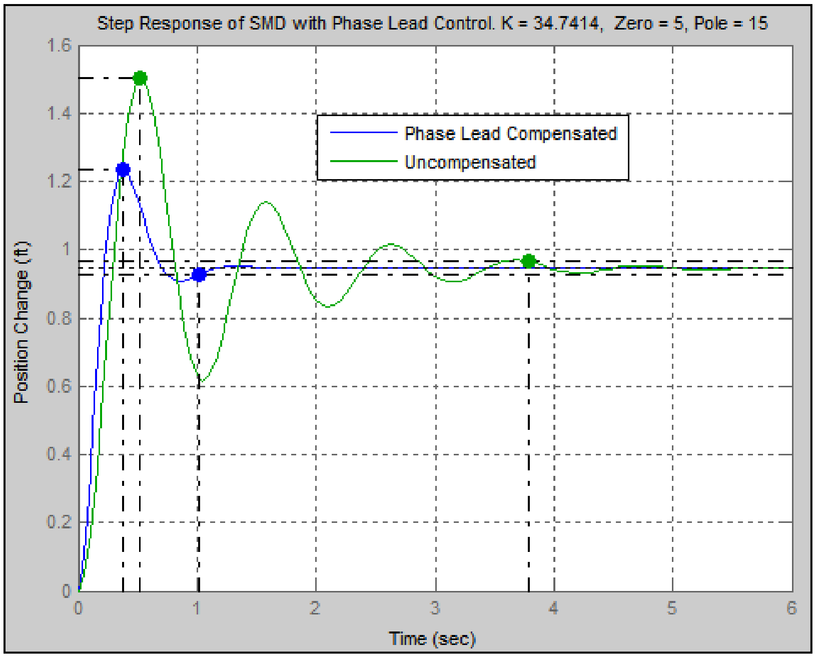
Try first $z = 5$ and $p = 15$, so the loop transfer function of the compensated system is $GH(s) = \dfrac{3K(s+5)}{(s+15)(s^2+2s+2)}$. ***Two questions must now be answered***. 1) Can roots be found with $\zeta > 0.5$? 2) Can large enough values for K be chosen so the steady-state error is small? In this case, the answer to both questions is "yes!" See the results below. If the answer to either question is "no", then slide the pole-zero combination along the axis to satisfy both requirements. Larger pole-zero separations can also be tried.

The root locus plot below is for the ***compensated system*** and the chosen pole locations correspond to $K \approx 34.74$. Note the compensator zero (also a zero of the closed loop system) is close to the complex poles, so we expect it to cause larger overshoots than would be expected for $\zeta = 0.5$.



Step 4: Check the step response.

The step response of the ***uncompensated system*** with gain $K \approx 34.74$ (to give the same steady-state error as the compensated system) ***shows a large overshoot*** $(\%OS \approx 59\%)$ and ***low damping*** with a settling time $T_s \approx 3.8$ (sec), while the step response of the ***compensated system*** shows a ***smaller overshoot*** ($\%OS \approx 30\%$) with ***higher damping*** with a settling time $T_s \approx 1$ (sec).



As an alternate design, the plots below show the root locus and step response plots for a phase lead compensator with $z = 3$ and $p = 25$. The root locus diagram shows the location of the closed loop poles for $K \approx 35$. The step response of the compensated system for $K \approx 35$ shows a 14% overshoot and a settling time of 0.7 seconds. Note the location of the real pole nearly cancels the compensators zero hence reducing its effect on the system overshoot.