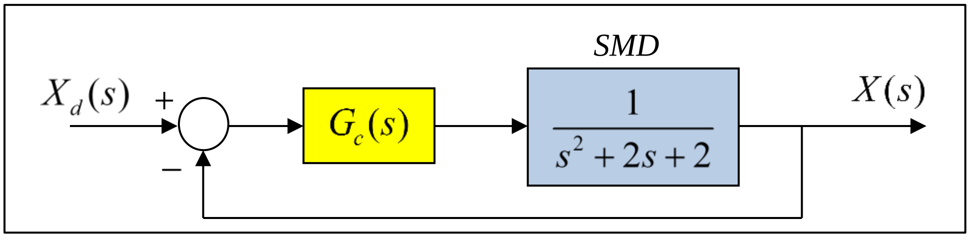
Introductory Motion and Control
Frequency Response Design of a Phase-Lag Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the ***frequency response design*** of a ***phase-lag compensator***, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the ***desired*** and ***actual*** positions of the mass.



Using proportional control ($G_c(s)=K$), ***large*** gains are required to control steady-state error to a step input. Unfortunately, large gains produce ***undesirable***, ***oscillatory*** closed-loop response. Below, a phase-lag compensator is designed to lower the steady-state error without introducing highly oscillatory behavior.

Problem: Design a phase-lag compensator so the closed-loop system has a ***steady-state position error*** $e_{ss}=1-x_{ss}<0.1$ to a unit step input and a ***phase margin*** $PM=45\text{ (deg)}$. Plot the step response of the resulting closed-loop system.

Frequency Response Design

Step 1: Determine the ***compensator gain*** required to satisfy the ***error specification.***

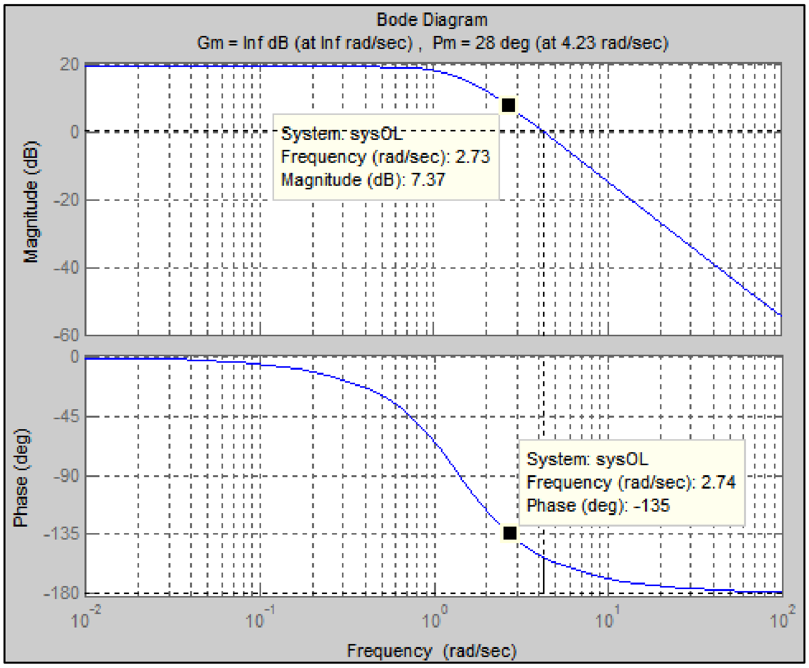
The steady state error can be defined in terms of the loop transfer function as

$$e_{ss}=\lim_{s\to 0}\left[\frac{1}{1+GH(s)}\right]=\frac{1}{1+K/2}=\frac{1}{1+K_p}<0.1$$

To meet the error specification, set $K_p>9$ and $K>18$.

Step 2: Evaluate the ***phase margin*** of the ***uncompensated system*** with $K=18$.

Using MATLAB, the phase margin of this (the uncompensated system) is $PM\cong 28\text{ (deg)}$. See plot below. This is well below the desired phase margin.



Phase Margin of Uncompensated System is 28 (deg) at 4.2 (rad/s)

Step 3: Locate the ***zero*** of the compensator

By examination of the Bode diagram, the phase margin requirement would be satisfied at 2.74 (rad/sec). This statement assumes the magnitude plot crosses over the ***zero-dB*** line at this point. So, ***locate the zero*** of the compensator ***at least one decade below this frequency***, for example, at 0.25 (rad/sec).

Step 4: Locate the ***pole*** of the compensator

To make 2.74 (rad/s) the zero-dB crossover point, require approximately ***8 dB of attenuation from the compensator***. (See Bode plot of uncompensated system.) Now calculate α by setting $-8 = 20\log(\alpha)$ to find $\alpha = 0.3981$. The first iteration yields the compensator

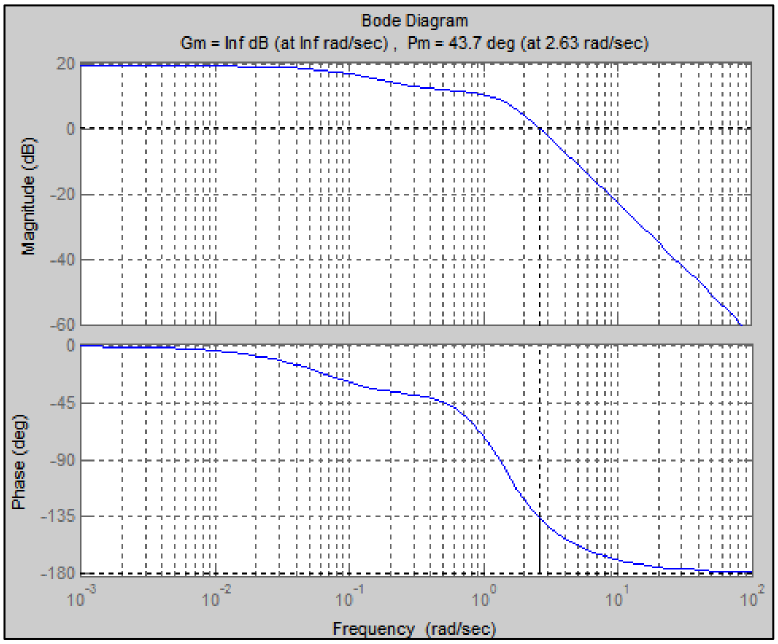
$$G_c(s) = 0.3981\left[\frac{s+0.25}{s+0.0995}\right] \qquad \text{(Phase-lag compensator)}$$

Step 5: Check the ***phase margin*** of the ***new compensated system.***

The Bode diagram of the loop transfer function of the compensated system

$$GH(s) = 0.3981\left[\frac{s+0.25}{s+0.0995}\right]\left[\frac{18}{s^2+2s+2}\right]$$

shows that the ***phase margin*** is $PM = 43.7$ (deg), satisfying the original requirement. Note the original gain of $K = 18$ is included in the plant transfer function.



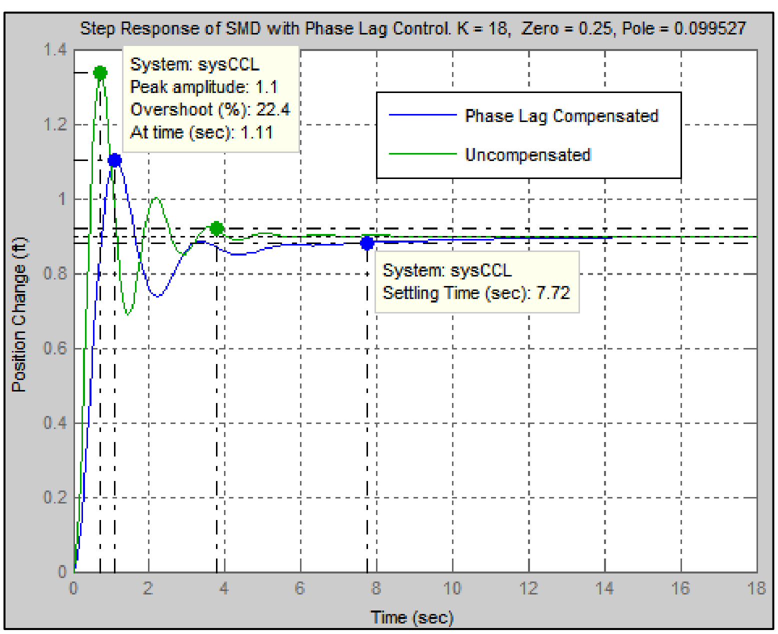
Phase Margin of Compensated System is 43.7 (deg)

Step 6: Repeat steps 3-5 until the ***desired phase margin*** is obtained.

The new compensated system nearly meets the specification. This result is used in the step response below.

Step 7: Check the step response.

Step response of the uncompensated system shows a large overshoot (about 49%) and low damping with a settling time of 3.78 seconds, while the step response of the phase-lag compensated system shows a smaller overshoot (around 22%) with higher damping and a longer settling time of 7.7 seconds. So, the reduction of overshoot comes at the price of increasing the settling time.



Step Response of Compensated System has ***less overshoot*** and ***more damping*** than the Uncompensated System.

Unfortunately, it also has a ***longer settling time***. This is typical of integrator-type compensators.

Frequency Response of the Compensated Closed Loop System:

The frequency response of the closed loop system is shown below. The ***bandwidth*** of the closed loop system is approximately 4 (rad/s).