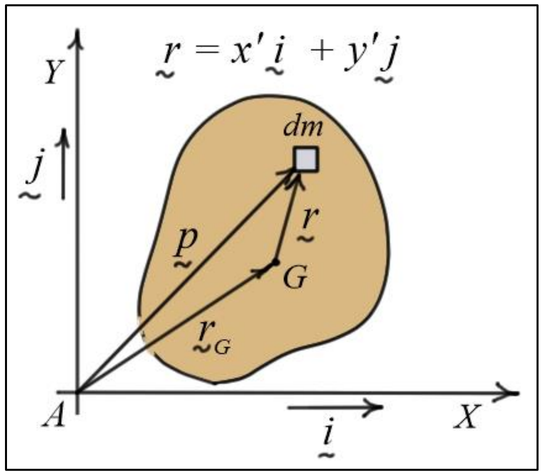
Elementary Dynamics
Center of Mass in Two Dimensions

Definition of Mass Center

The figure depicts a rigid body in two dimensions. The mass center of the body is defined as that point where



$$\boxed{\int_B \underset{\sim}{r}\, dm = 0} \quad \Rightarrow \begin{cases} \displaystyle\int_B x'\, dm = 0 \\ \displaystyle\int_B y'\, dm = 0 \end{cases}$$

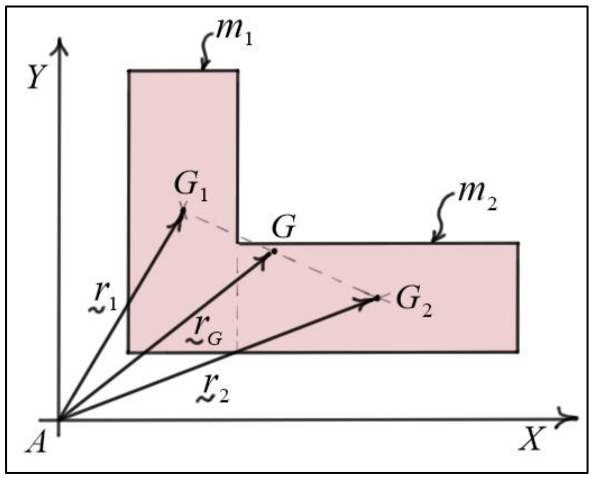
A more ***practical definition*** of the location of the mass center of a body can be developed as follows.

$$\int_B \underset{\sim}{p}\, dm = \int_B \left(\underset{\sim}{r}_G + \underset{\sim}{r}\right) dm = \underbrace{\left(\int_B dm\right)}_{M} \underset{\sim}{r}_G + \underbrace{\int_B \underset{\sim}{r}\, dm}_{\text{zero by definition of mass center}} = M\, \underset{\sim}{r}_G \quad \Rightarrow \boxed{\underset{\sim}{r}_G = \frac{1}{M}\int_B \underset{\sim}{p}\, dm}$$

This result allows calculation of the mass center location relative to an arbitrary (convenient) point A.

Mass Center – Composite Shapes

Bodies having complex shapes can often be subdivided into bodies with more convenient (simpler) shapes. For example, the L-shaped bracket shown in the figure can be subdivided into two rectangles. In cases like this, ***tables*** can be used to determine the mass center locations of each of the common geometric shapes. That information is then used to compute the location of the mass center of the composite shape as follows.



$$\boxed{\underset{\sim}{r}_G = \frac{\sum_i m_i \underset{\sim}{r}_i}{\sum_i m_i}}$$

For the L-shaped bracket, $\boxed{\underset{\sim}{r}_G = \left(m_1 \underset{\sim}{r}_1 + m_2 \underset{\sim}{r}_2\right) \big/ \left(m_1 + m_2\right)}$. Note that G may or may not be located on the physical body.