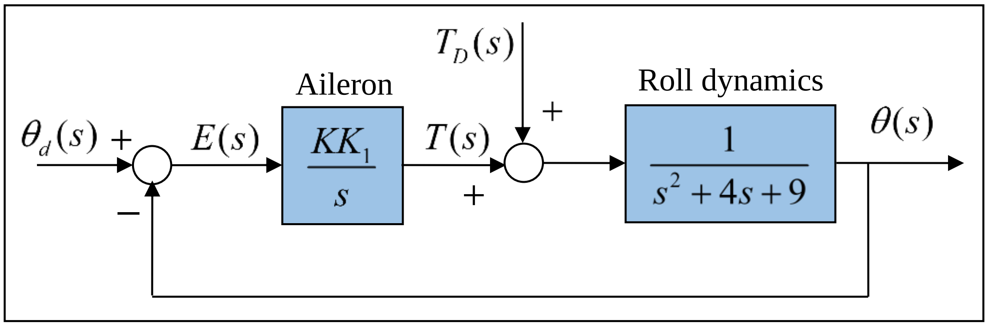
Introductory Control Systems
Design Problem – Roll Control of a Small Aircraft
Ref: Dorf & Bishop, Modern Control Systems, 12th edition, Prentice-Hall, Inc, 2010

The block diagram of the roll control system of a small aircraft is shown below.

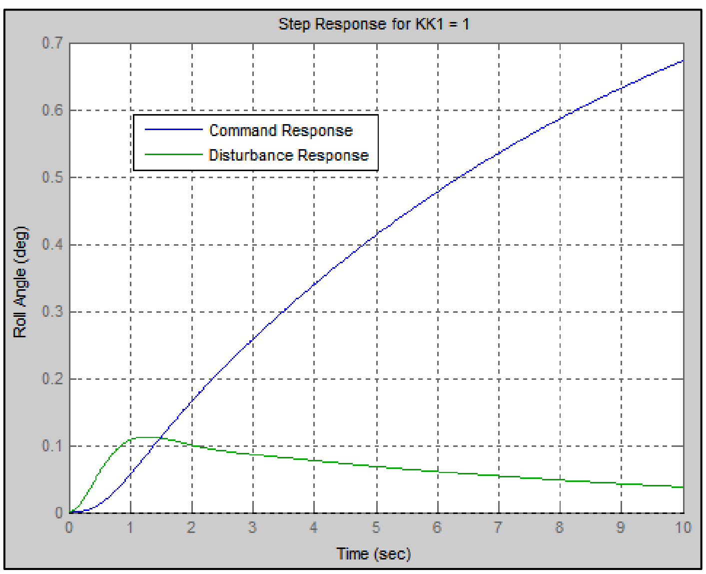


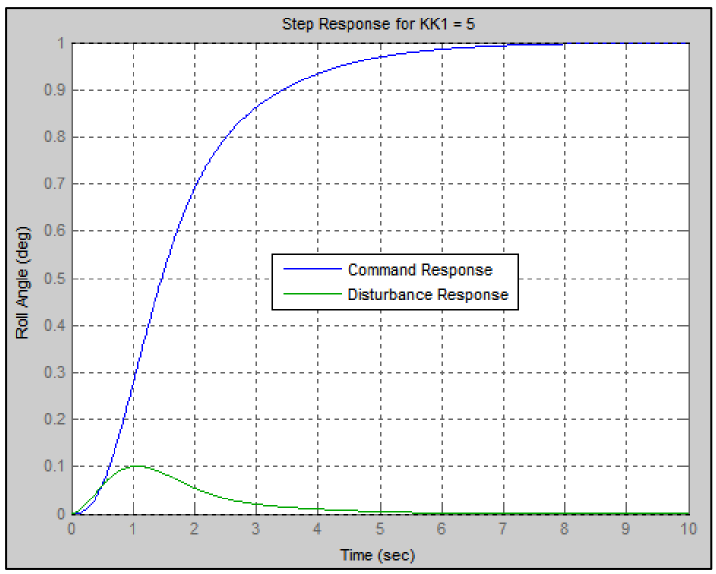
The variables $\theta_d(s)$ and $\theta(s)$ represent the ***desired*** and ***actual*** roll angles of the aircraft, and the variable $T_d(s)$ represents a **disturbance roll torque** on the aircraft. **Question**: Are there ***reasonable values*** of the ***product*** KK_1 that allows the system to have a ***desirable response*** to step commands in roll angle and to maintain ***small roll angles*** in the presence of a disturbance?

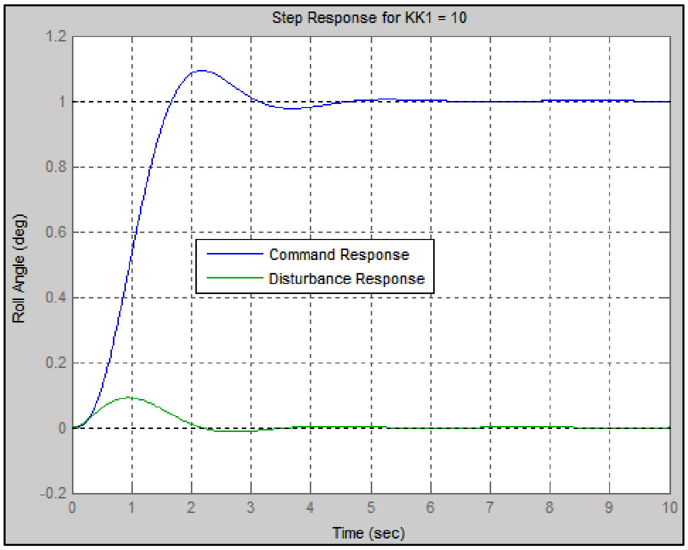
Using block diagram reduction, the ***closed-loop*** and ***disturbance*** transfer functions are

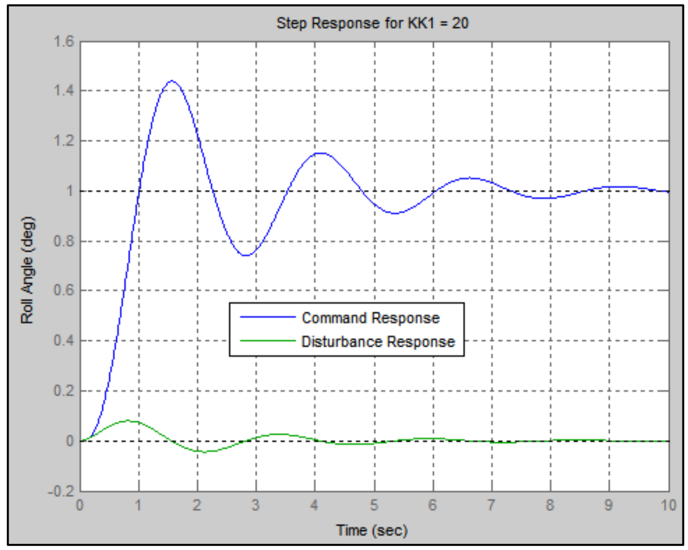
$$\frac{\theta}{\theta_d}(s) = \frac{KK_1}{s^3+4s^2+9s+KK_1} \qquad \frac{\theta}{T_d}(s) = \frac{s}{s^3+4s^2+9s+KK_1}$$

From these transfer functions, it is clear the system has ***no steady-state error due to a step command*** in the desired roll angle, and that there is ***no steady-state error due to a step input disturbance***. These are both desirable characteristics. The step responses of the system for various KK_1 values are shown below.









The response for $KK_1 = 1$ is not good. After ten seconds, the system has achieved only 70% of its final value. The response for $KK_1 = 5$ is good. The response is over-damped and is close to its final value (settling time) in about six seconds. The response for $KK_1 = 10$ is close to its final value in less than five seconds, but the response is under-damped with about a 10% overshoot. If an over-damped or critically damped response is desired, there may be better choices for KK_1 between five and ten. Finally, the response for $KK_1 = 20$ is not good. The response is under-damped with more than 40% overshoot, and the system does not get close to its final value until beyond eight seconds.

Summary: This system has over-damped response for smaller values of KK_1 and under-damped response for larger values of KK_1. The settling times of the system are larger for both smaller and larger KK_1 values. Smaller settling times are found for intermediate values.

Steady-State Error Due to a Ramp Input

The ***steady state error*** of the system due to a ***ramp input*** can be calculated using the final value theorem. Using the block diagram, the ***error transfer function*** is

$$\boxed{\frac{E}{\theta_d}(s) = \frac{s(s^2+4s+9)}{s^3+4s^2+9s+KK_1}}$$

Using the ***final value theorem***, the steady-state error due to a ramp input is

$$\boxed{e_{ss} = \lim_{s\to 0}\left(\not{s}\cdot\frac{1}{s^{\not{2}}}\cdot\frac{E}{\theta_d}(s)\right) = \lim_{s\to 0}\left(\frac{1}{\not{s}}\cdot\frac{\not{s}(s^2+4s+9)}{s^3+4s^2+9s+KK_1}\right) = \frac{9}{KK_1}}$$

So, the steady-state error for a ramp input can be ***decreased*** by ***increasing*** the value of KK_1.

The roll ramp response of the system for various KK_1 values is shown in the figure below. The black line represents the unit ramp (slope equal to one) for comparison with the response plots. Note that all the responses eventually become parallel to the unit ramp. The vertical distance from the response plot to the ramp represents the steady-state roll angle error. For example, note that the steady-state error for $KK_1 = 10$ is equal to 0.9. This value is evident in the upper right corner of the plot. At $t = 10$ (sec), the ramp value is ten, but the response value (red line) is just over nine.