Introductory Motion and Control
Compensator Design for ITAE Optimal Response with Pre-filters
(Reference: Dorf & Bishop, Modern Control Systems, Prentice-Hall, 2001)

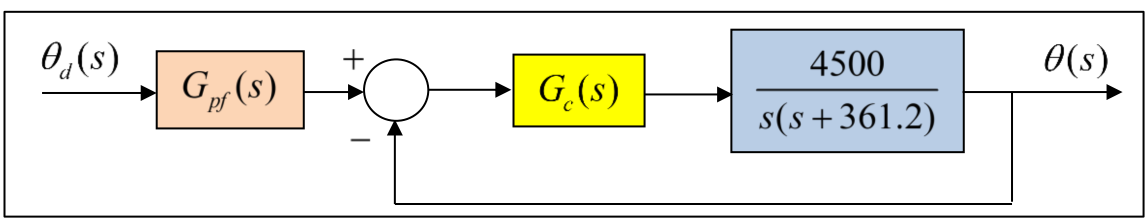
The forms of ***ITAE optimal*** transfer functions for ***step*** and ***ramp*** inputs are given in Tables 5.6 and 5.7 of Dorf & Bishop's Modern Control Systems. To guarantee ITAE optimal behavior, a compensator can be designed to force the closed-loop transfer function to be of ***ITAE optimal form***. The use of ***pre-filters*** may be required to obtain optimal form.

Design Strategy

1. ***Step Input***: Assume $\zeta = 0.7$; ***Ramp Input***: Assume $\zeta = 1.6$
2. Then ***select*** ω_n to satisfy the ***settling time*** specification.
3. Find the compensator parameters to give a ***characteristic equation*** of the form given in Table 5.6 (step input) or Table 5.7 (ramp input).
4. Design a ***pre-filter*** to "cancel" the unwanted terms in the numerator of the closed-loop transfer function.

Example

Design a PID compensator that will produce ***ITAE optimal, closed-loop step response*** with a settling time $T_s = 0.005\ \text{(sec)}$ for the aircraft attitude control system shown in the block diagram. Use a pre-filter as necessary.



Solution

1. Find ω_n by setting $T_s = 4/\zeta\omega_n = 0.005\ \text{(sec)}$. Assuming $\zeta = 0.7$ (for a step input), $\omega_n = 1143\ \text{(rad/sec)}$.
2. Assuming a PID controller of the form $G_c(s) = \dfrac{K(s^2 + as + b)}{s}$, the closed loop transfer function (without the pre-filter) is found to be

$$T(s) = \frac{4500K(s^2 + as + b)}{s^2(s + 361.2) + 4500K(s^2 + as + b)}$$

With ***PID*** control and ***no pre-filter***, this system is ***type-2***, so it can be ***optimized*** for ***ramp input***. However, in this example, the ***optimization*** is for ***step response***.

3. Equate the ***characteristic equation*** with that provided by Table 5.6 with $\omega_n = 1143$.

$$\boxed{\begin{array}{c} s^3 + (361.2 + 4500K)s^2 + (4500aK)s + (4500bK) \\ = s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3 \end{array}}$$

Equating the polynomial coefficients and solving for K, a, and b gives

$$K = 0.36423 \qquad a = 1713.73 \qquad b = 9.110673 \times 10^5$$

or

$$K_p = 624.2 \qquad K_I = 331{,}838 \qquad K_D = 0.36423$$

4. ***Pre-filter***: To provide ITAE optimal response, choose the pre-filter

$$\boxed{G_{pf}(s) = \frac{(1143)^3}{1639(s^2 + as + b)}}$$

So, the final transfer function has ***ITAE optimal form***

$$\frac{\theta_d(s)}{\theta(s)} = \left(\frac{(1143)^3}{1639(s^2 + as + b)}\right)\left(\frac{1639(s^2 + as + b)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + (1143)^3}\right)$$

$$\Rightarrow \quad \boxed{\frac{\theta_d(s)}{\theta(s)} = \frac{(1143)^3}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + (1143)^3}}$$

5. Closed-loop step response ***with*** and ***without*** pre-filter: From the plots below, it is clear the closed loop system ***without a pre-filter*** has ***large overshoot*** due to the zeros in the PID compensator. This problem is removed by including a pre-filter. Both systems have settling times slightly larger than 0.005 (sec). Further increases on the choice of ω_n will resolve this issue as well.

6. Error analysis ***with a pre-filter***: The system error can be calculated as follows

$$E(s) = \theta_d(s) - \theta(s) = \theta_d(s) - \left(G_{pf}(s)T(s)\right)\theta_d(s) = \left(1 - G_{pf}(s)T(s)\right)\theta_d(s)$$

$$= \left(1 - \frac{(1143)^3}{s^3 + 2000s^2 + (2.8089E6)s + (1143)^3}\right)\theta_d(s)$$

So, the error transfer function is

$$\frac{E(s)}{\theta_d(s)} = \frac{s(s^2 + 2000s + 2.8089E6)}{s^3 + 2000s^2 + (2.8089E6)s + (1143)^3}$$

Note the system ***with the pre-filter*** has a ***zero*** steady-state error to a ***step input*** and a ***finite*** steady-state error to a ***ramp input*** (typical of a ***type-1*** system).

7. Error analysis ***with no pre-filter***: The system with no pre-filter is a ***type-2*** system, so it has a ***zero*** steady-state error to both ***step*** and ***ramp*** inputs. Clearly, ***some error control is lost when using the pre-filter***.