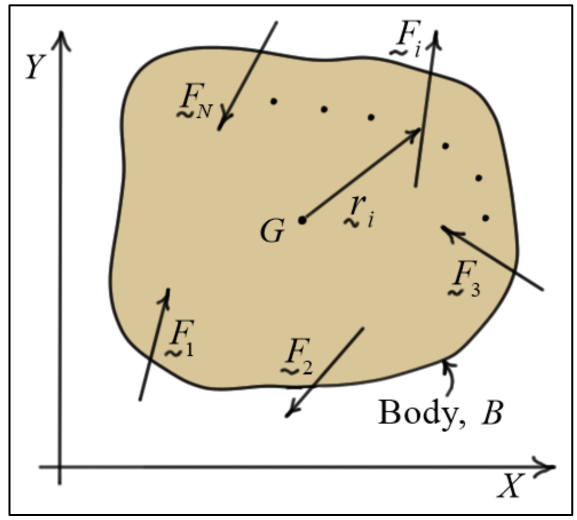
Elementary Dynamics
Newton's Law for Rigid Body Motion in Two Dimensions

General Plane Motion

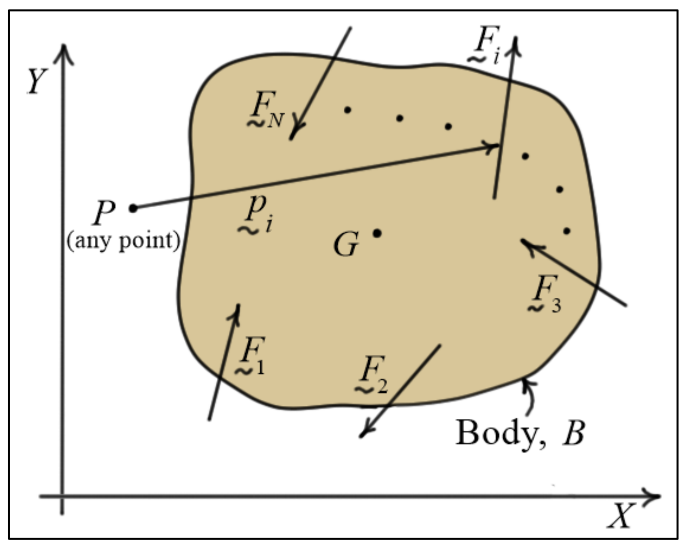
The figure depicts a rigid body moving in two dimensions. The motion is caused by a series of N forces $\underset{\sim}{F}_i\ (i=1,\ldots,N)$. Generally, each force has the effect of both translating and rotating the body. Newton's laws of translational and rotational motion are



$$\boxed{\begin{aligned}&\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G\\&\sum_i (\underset{\sim}{M}_G)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = I_G\,\underset{\sim}{\alpha}\end{aligned}}$$

Here, $(\underset{\sim}{M}_G)_i$ represents the ***moment*** of force $\underset{\sim}{F}_i$ about the ***mass center*** G, and I_G represents the ***moment of inertia*** of the body about a Z axis passing through G.

Note the equation for rotational motion as written in the boxed equation requires that moments be taken about the mass center G. If moments are to be taken about any point P other than G, Newton's laws of translational and rotational motion are written as



$$\boxed{\begin{aligned}&\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G\\&\sum_i (\underset{\sim}{M}_P)_i = \sum_i (\underset{\sim}{p}_i \times \underset{\sim}{F}_i) = I_G\,\underset{\sim}{\alpha} + \left(\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G\right)\end{aligned}}$$

Note the term $\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G$ is added to the right side of the moment equation and represents the moment of vector $m\underset{\sim}{a}_G$ about point P. The line of action of $m\underset{\sim}{a}_G$ is assumed to pass through G.

Special Cases

Pure Translational Motion

The equations of motion of a rigid body undergoing pure translational motion can be written as follows.

$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G \qquad \sum_i (\underset{\sim}{M}_G)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = 0$$

or

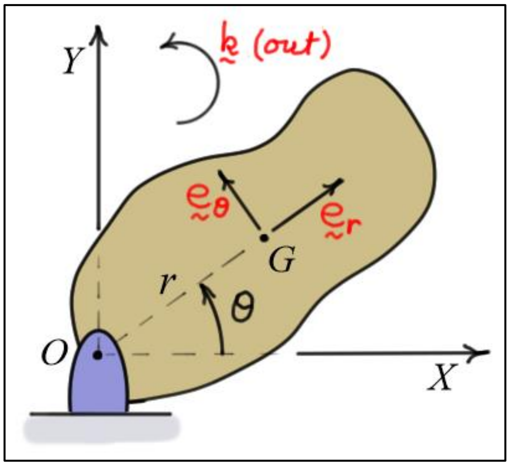
$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G \qquad \sum_i (\underset{\sim}{M}_P)_i = \sum_i (\underset{\sim}{p}_i \times \underset{\sim}{F}_i) = \left(\underset{\sim}{r}_{G/P} \times m\underset{\sim}{a}_G\right)$$

Fixed Axis Rotation

When a body is undergoing fixed axis rotation as shown in the figure, the equations of motion can be written as

$$\sum_i \underset{\sim}{F}_i = m\underset{\sim}{a}_G = m\left(r\alpha\underset{\sim}{e}_\theta - r\omega^2\underset{\sim}{e}_r\right) = m\left(r\ddot{\theta}\underset{\sim}{e}_\theta - r\dot{\theta}^2\underset{\sim}{e}_r\right)$$

$$\sum_i (\underset{\sim}{M}_O)_i = \sum_i (\underset{\sim}{r}_i \times \underset{\sim}{F}_i) = I_O\,\underset{\sim}{\alpha} = I_O\,\ddot{\theta}\,\underset{\sim}{k}$$



Here I_O represents the ***moment of inertia*** of the body about a Z axis passing through the ***fixed-point*** O.